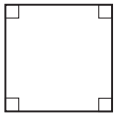
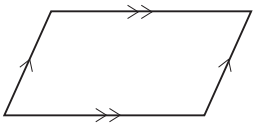
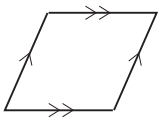
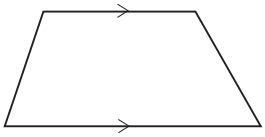
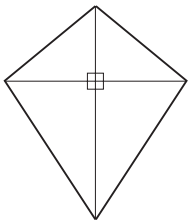
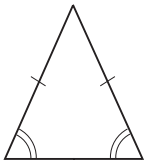
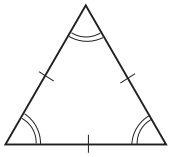
9 Area, Perimeter and Volume

9.1 2-D Shapes

The following table gives the names of some 2-D shapes. In this section we will consider the properties of some of these shapes.

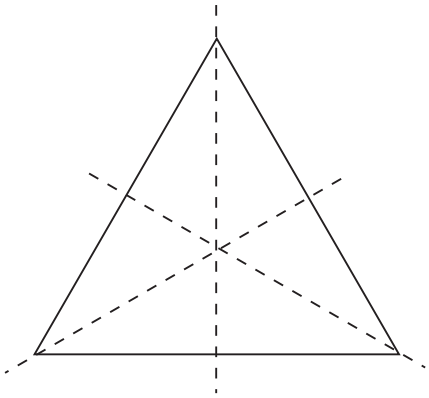
Rectangle		All angles are right angles ($90°$) Opposite sides have the same length
Square		All the sides have the same length All angles are right angles ($90°$)
Parallelogram		Opposite sides have the same length
Rhombus		All the sides have the same length Diagonals bisect at right angles
Trapezium		
Kite		Diagonals intersect at right angles
Isosceles Triangle		Two sides have the same length and the angles opposite these two sides are equal
Equilateral Triangle		All angles are $60°$

Example 1

Draw the lines of symmetry of an equilateral triangle.

Solution

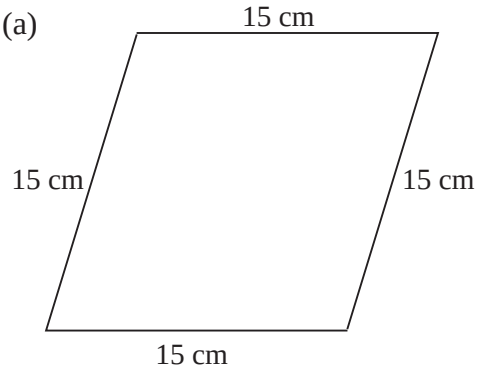
There are 3 lines of symmetry, as shown in the diagram. They join each vertex (corner) to the midpoint of the opposite side.



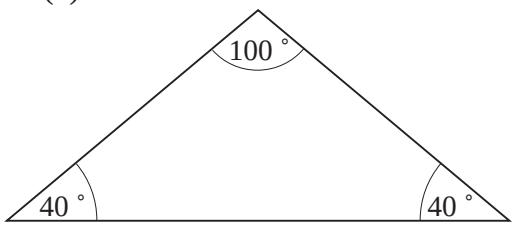
Example 2

Name each of the following shapes:

(a)



(b)



Solution

(a) This is a *rhombus* because all the sides have the *same* lengths.

(b) This is an *isosceles triangle* because two of the angles are the same size.

Example 3

State the order of rotational symmetry of:

(a) a *trapezium*,

(b) a *parallelogram.*

Solution

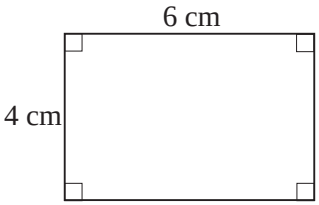
(a) 1

(b) 2 (unless the parallelogram happens to be a square, in which case the order of rotational symmetry would be 4).

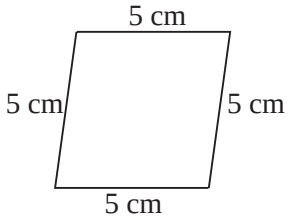
Exercises

1. Name each of the following shapes:

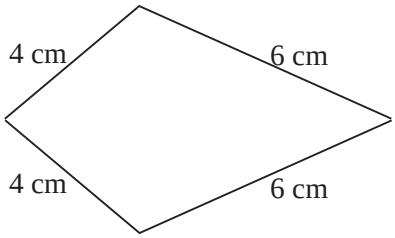
(a)



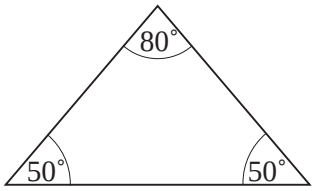
(b)



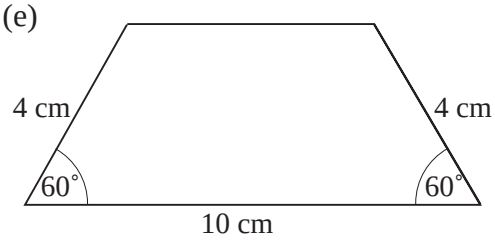
(c)



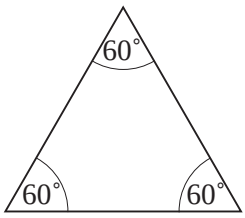
(d)



(e)



(f)



2. Draw diagrams to show the lines of symmetry of:

(a) a *kite,* (b) a *square,*

(c) a *rectangle,* (d) an *isosceles triangle.*

3. How many lines of symmetry are there for:

(a) a *parallelogram,* (b) a *rhombus* ?

4. State whether each of the following statements is *true* or *false.*

(a) A *square* is also a *rhombus.*

(b) A *square* is also a *kite.*

(c) A *rectangle* is also a *kite.*

(d) A *parallelogram* is also a *kite.*

(e) A *rectangle* is also a *parallelogram.*

5. Write down the order of rotational symmetry of:

(a) a *rhombus,* (b) a *square,*

(c) an *isosceles triangle,* (d) an *equilateral triangle,*

(e) a *kite.*

6. A triangle has *one* line of symmetry. What type of triangle is it?

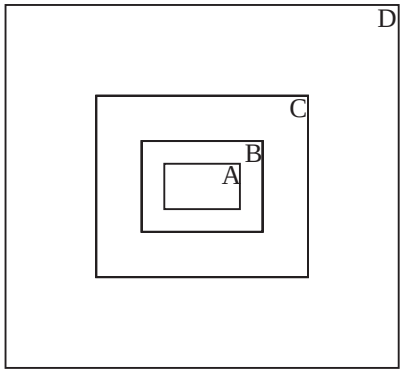
7. Draw a trapezium that has:
   (a) *one* line of symmetry, (b) *no* lines of symmetry.

8. A right-angled triangle is also an *isosceles* triangle. What sizes are the other angles in this triangle?

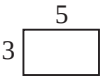
9. For a semicircle:
   (a) draw a diagram to show its lines of symmetry,
   (b) state its order of rotational symmetry.

10. (a) Draw a diagram to show the lines of symmetry of a *regular pentagon*.
    (b) State the order of rotational symmetry of a *regular octagon*.

11. Rosemary drew these rectangles using a computer:



Rectangle A has *width 3* and *length 5*:



The computer repeated these instructions to draw the other rectangles:

new *width* = previous *width* × 2

new *length* = previous *length* + previous *width*

Copy and complete this table.

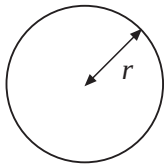
	width	*length*
rectangle A	3	5
rectangle B	.....	.....
rectangle C	.....	.....
rectangle D	.....	.....

(KS3/94/Ma/3-5/P1)

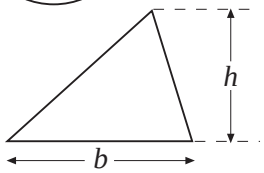
9.2 Area of Special Shapes

In this section we calculate the area of various shapes.

Area of a circle $= \pi r^2$

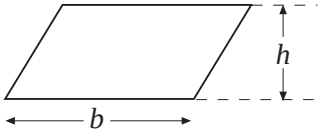


Area of a triangle $= \frac{1}{2} b h$



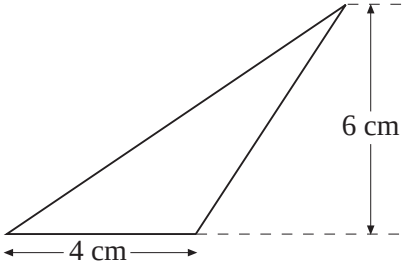
(h is perpendicular height)

Area of a parallelogram $= b h$



Example 1

Calculate the *area* of the triangle shown.



Solution

$$\text{Area} = \frac{1}{2} \times 4 \times 6$$

$$= 12 \text{ cm}^2$$

Example 2

Calculate the *area o*f a circle with diameter 10 m.

Solution

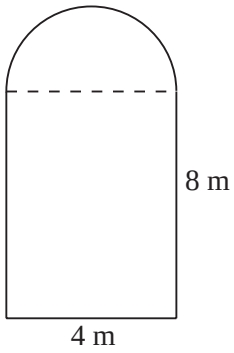
Radius $= 10 \div 2 = 5$ m

Area $= \pi \times 5^2 = 78.53981634 \text{ m}^2$

$= 78.5 \text{ m}^2$ (to 3 significant figures)

Example 3

Calculate the *area* of the shape shown:



Solution

Area of rectangle $= 4 \times 8$

$= 32 \text{ m}^2$

Radius of semicircle $= 4 \div 2 = 2 \text{ m}$

Area of semicircle $= \frac{1}{2} \times \pi \times 2^2$

$= 6.283185307 \text{ m}^2$

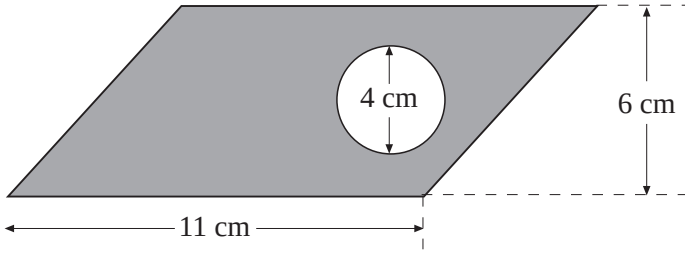
Total area $= 32 + 6.283185307 = 38.283185307 \text{ m}^2$

$= 38.3 \text{ m}^2$ (to 3 significant figures)

Example 4

The diagram shows a piece of card in the shape of a parallelogram, that has had a circular hole cut in it.

Calculate the area of the shaded part.



Solution

Area of parallelogram $= 11 \times 6$

$= 66 \text{ cm}^2$

Radius of circle $= 4 \div 2 = 2 \text{ cm}$

Area of circle $= \pi \times 2^2$

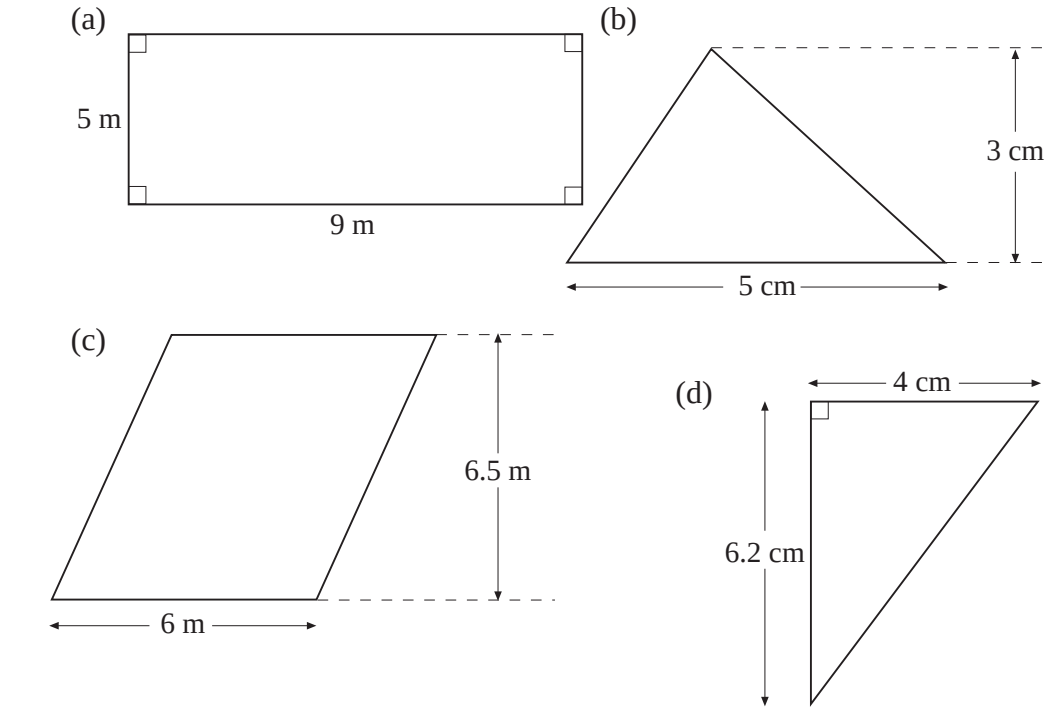
$= 12.56637061 \text{ cm}^2$

Area of shape $= 66 - 12.56637061 = 53.43362939 \text{ cm}^2$

$= 53.4 \text{ cm}^2$ (to 3 significant figures)

Exercises

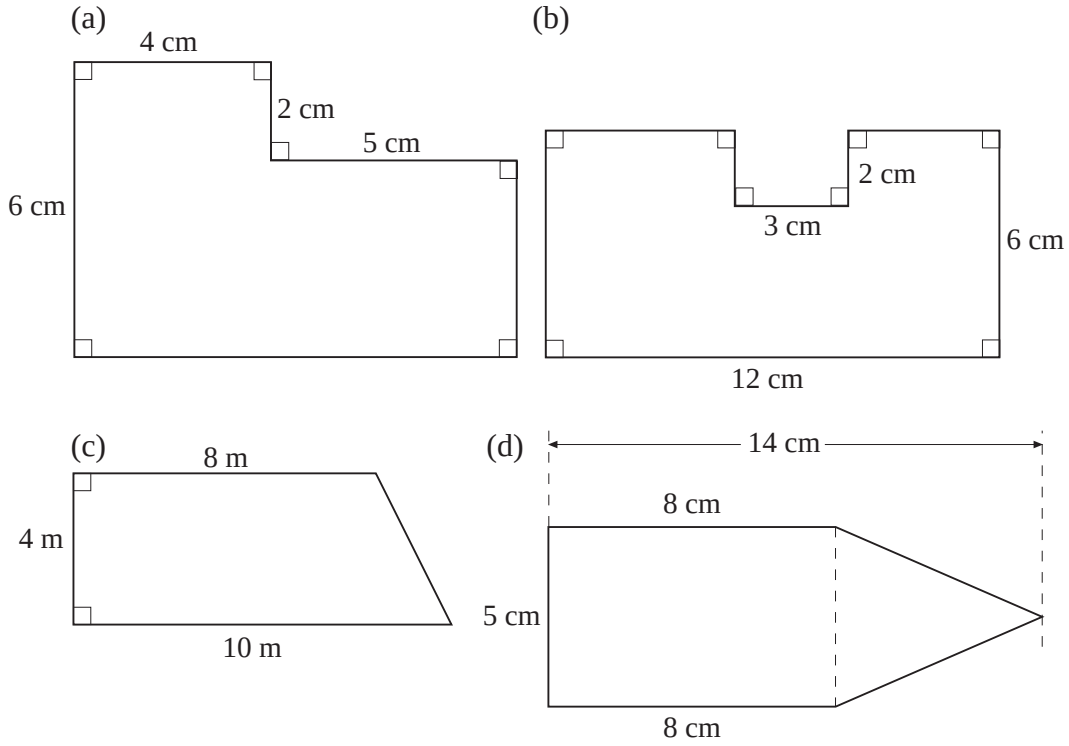
1. Calculate the area of each of the following shapes:



2. Calculate, giving your answers correct to 3 significant figures, the *area* of a circle with:

   (a) radius 6 m, (b) diameter 20 cm, (c) diameter 9 cm.

3. Calculate the *area* of each of the following shapes, giving your answers correct to 3 significant figures:

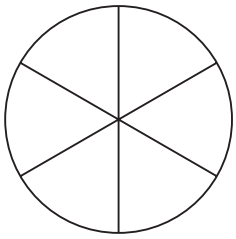


4. Calculate, giving your answers correct to 3 significant figures, the *area* of the semicircle with:

   (a) radius 30 cm, (b) diameter 14 mm.

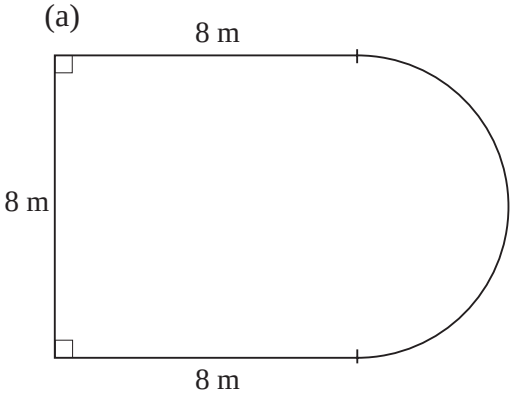
5. A circle of radius 8 cm is cut into 6 parts of equal size, as shown in the diagram.

   Calculate the *area* of each part, giving your answer correct to 2 decimal places.

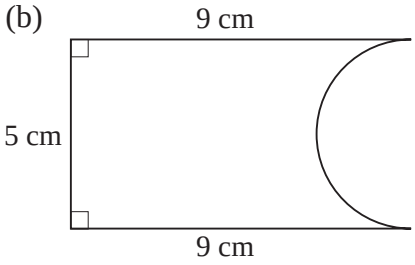


6. Giving your answers correct to 3 significant figures, calculate the *area* of each of the following shapes. Each of the curved parts is a semicircle.

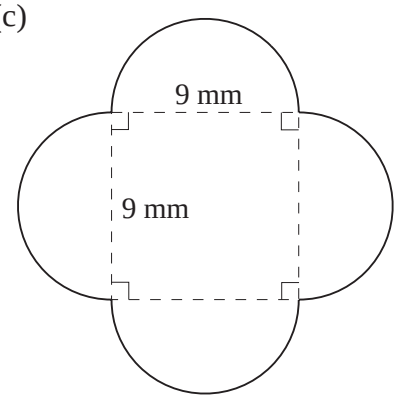
(a)



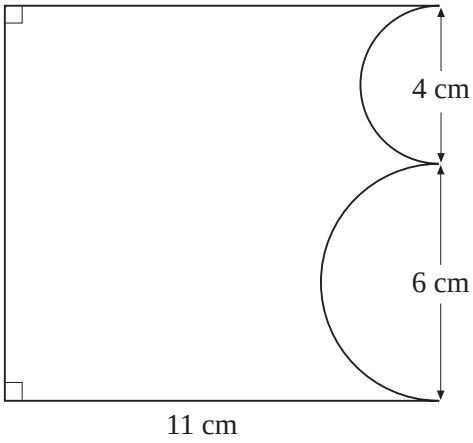
(b)



(c)

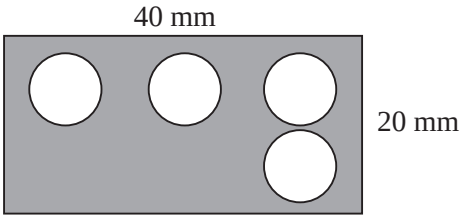


(d)

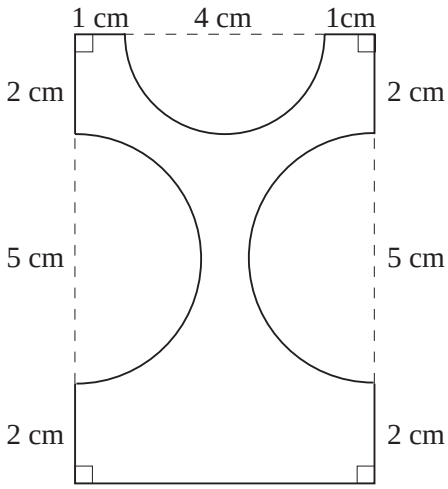


7. A rectangular metal plate is shown in the diagram. Four holes of diameter 8 mm are drilled in the plate.

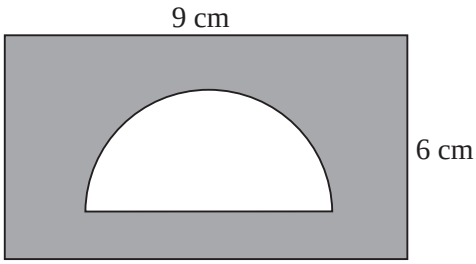
   Calculate the *area* of the remaining metal, giving your answer correct to 2 decimal places.



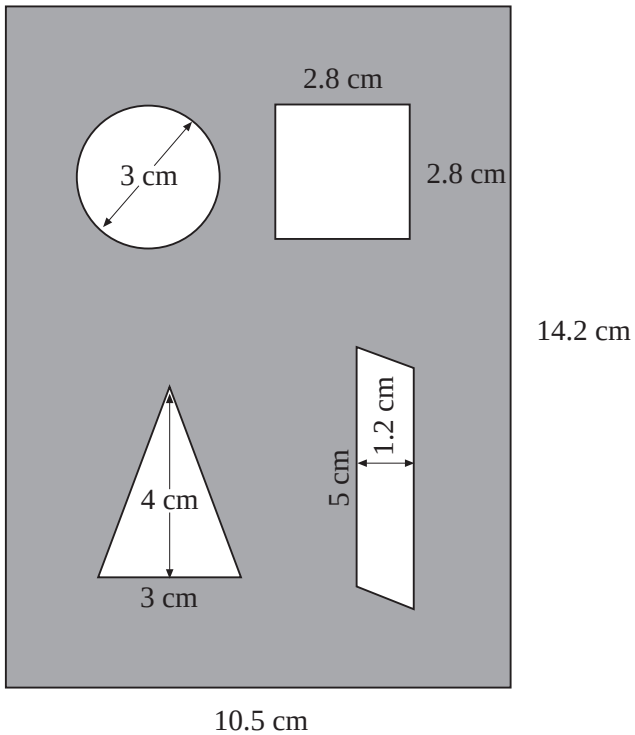
8. Calculate the *area* of the shape shown, giving your answer correct to 1 decimal place.



9. The area that has been shaded in the diagram has an area of $21.8\ \text{cm}^2$. Calculate the *diameter* of the semi-circular hole, giving your answer to the nearest millimetre.

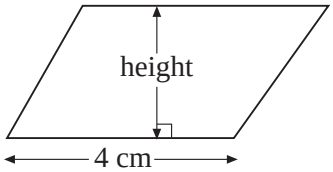


10. The diagram shows the lid of a child's shape-sorter box. Calculate the *area* of the lid, giving your answer correct to 1 decimal place.

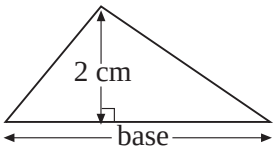


11. Each shape in this question has an *area* of 10 cm^2.
No diagram is drawn to scale.

(a) Calculate the height of the parallelogram.

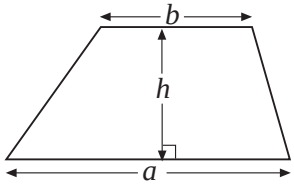


(b) Calculate the length of the base of the triangle.

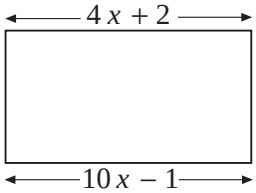


(c) What might be the values of *h*, *a* and *b* in this trapezium?

What else might be the values of *h*, *a* and *b* ?



(d) Look at this rectangle:

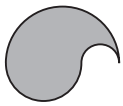


area = 10 cm^2

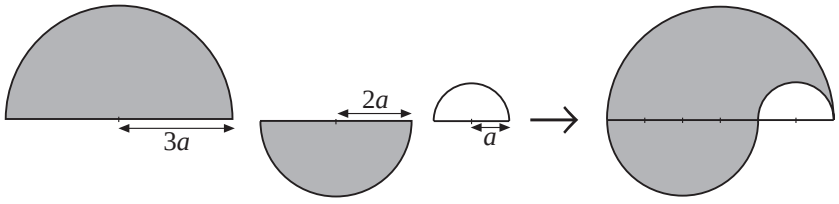
Calculate the value of x and use it to find the length and width of the rectangle.

Show your working.

(KS3/98/Ma/Tier 5-7/P1)

12. This shape is designed using 3 semi-circles. 

The radii of the semi-circles are $3a$, $2a$ and a.



(a) Find the area of each semi-circle, in terms of a and π, and show that the *total* area of the shape is $6\pi a^2$.

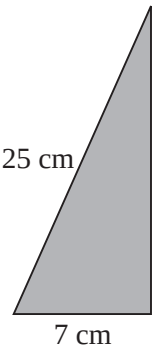
(b) The area, $6\pi a^2$, of the shape is 12 cm^2.

Write an equation in the form $a = \ldots$, leaving your answer in terms of π.

Show your working and *simplify* your equation.

(KS3/98/Ma/Tier 6-8/P1)

13. Calculate the area of this triangle.

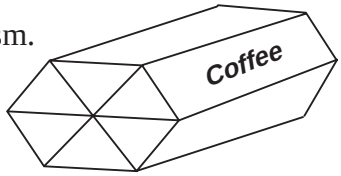


NOT
TO
SCALE

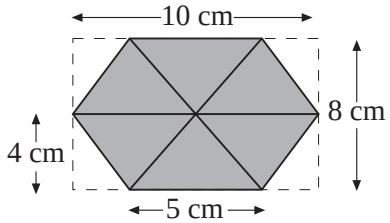
Show your working.

(KS3/97/Ma/Tier 5-7/P2)

14. A box for coffee is in the shape of a hexagonal prism.



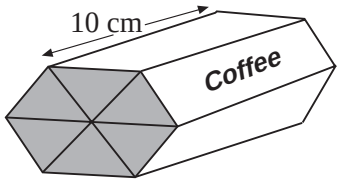
One end of the box is shown below.



NOT
TO
SCALE

Each of the 6 triangles in the hexagon has the same dimensions.

(a) Calculate the total *area* of the hexagon.
Show your working.



(b) The box is 10 cm long.

After packing, the coffee fills 80% of the box.

How many grams of coffee are in the box?

(The mass of $1\ \text{cm}^3$ of coffee is 0.5 grams.)

Show your working.

(c) A 227 g packet of the same coffee costs £2.19.
How much per 100 g of coffee is this?
Show your working

(KS3/98/Ma/Tier 5-7/P2)

9.3 Perimeter of Special Shapes

In this section we calculate the perimeters of various shapes. The perimeter of a circle is referred to as the 'circumference'.

> The circumference, C, of a circle $= 2\pi r$ or πd where r is the radius and d is the diameter of the circle.

Example 1

Calculate the circumference of a circle with radius 8 cm.

Solution

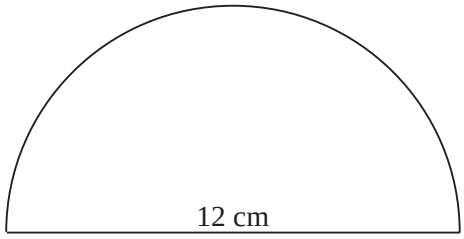
Using the formula, $C = 2\pi r$, gives

$C = 2 \times \pi \times 8 = 50.26548246$ cm

$= 50.3$ cm (to 3 significant figures)

Example 2

The diagram shows a semicircle of diameter 12 cm.
Calculate the perimeter of the semicircle.



Solution

Length of curve $= \pi \times 12 \div 2$

$= 18.84955592$ cm

Straight edge $= 12$ cm

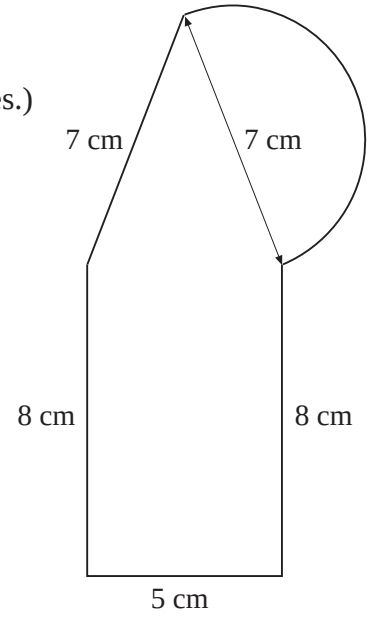
Total perimeter $= 12 + 18.84955592$

$= 30.84955592$ cm

$= 30.8$ cm (to 3 significant figures.)

Example 3

The diagram shows a shape that is made up of a rectangle, a triangle and a semicircle.
Calculate its perimeter.



Solution

Length of curve $= \pi \times 7 \div 2$

$= 10.99557429$ cm

Total perimeter $= 8 + 5 + 8 + 7 + 10.99557429$

$= 38.99557429$ cm

$= 39.0$ cm (to 3 significant figures)

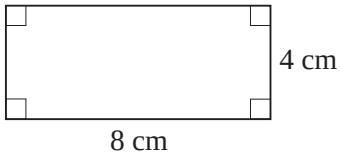
Exercises

1. Giving your answers correct to 3 significant figures, calculate the *circumference* of a circle with:

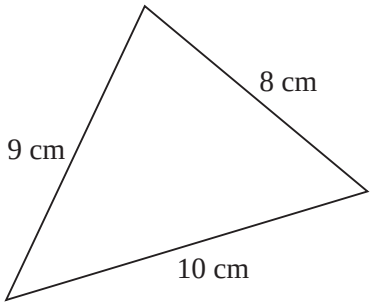
   (a) radius 6 m, (b) diameter 15 cm, (c) radius 8 mm.

2. Calculate the *perimeter* of each of the following shapes:

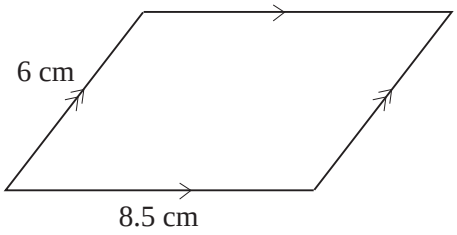
   (a)



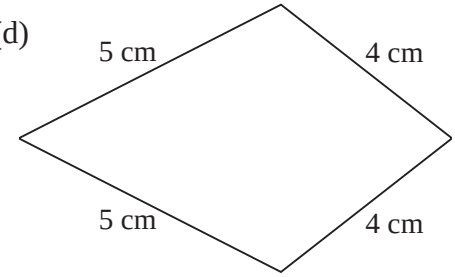
   (b)



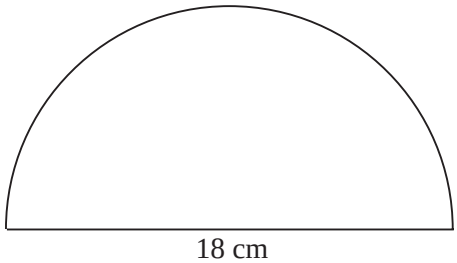
   (c)



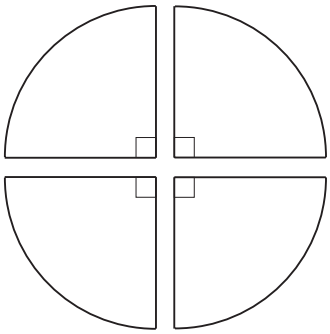
   (d)



3. Giving your answer correct to 3 significant figures, calculate the *perimeter* of the semicircle shown.



4. A circle of radius 8 cm is cut into four equal parts as shown in the diagram:

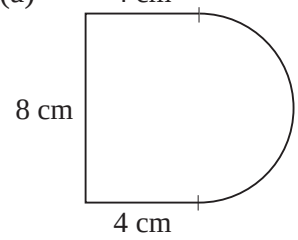


(a) Calculate the *circumference* of the original circle, giving your answer correct to 2 decimal places.

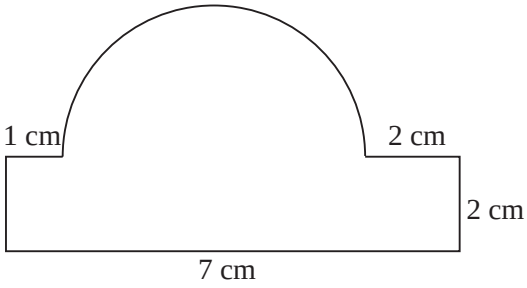
(b) Calculate the *perimeter* of each of the 4 parts, giving your answers correct to 2 decimal places.

5. Calculate the *perimeter* of each of the following shapes, giving your answers correct to 1 decimal place. The circular parts are either semicircles or quarters of circles.

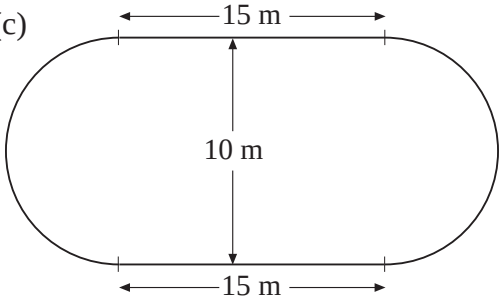
(a)



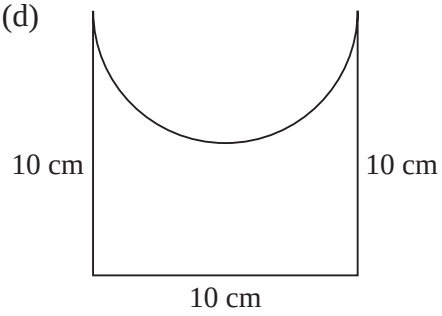
(b)



(c)

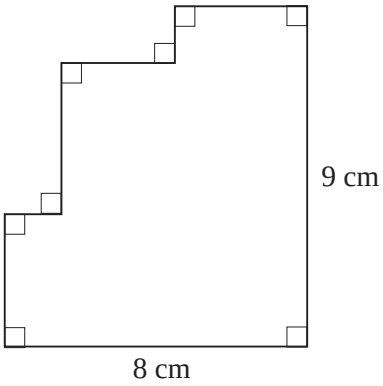


(d)

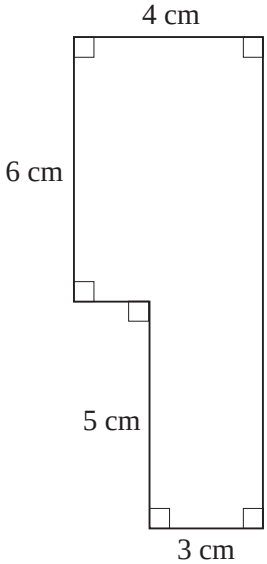


6. Calculate the *perimeter* of each of the following shapes:

(a)

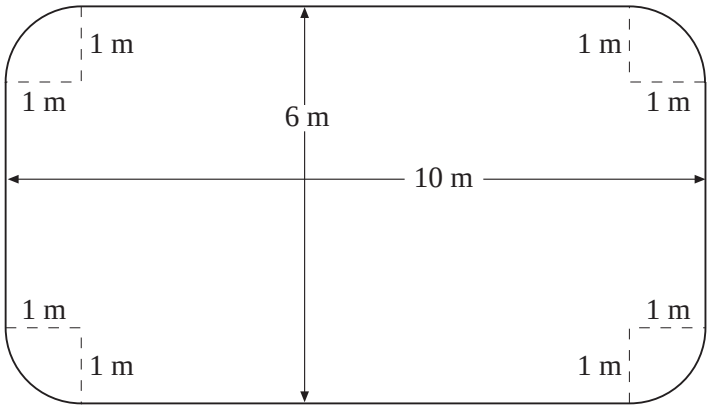


(b)

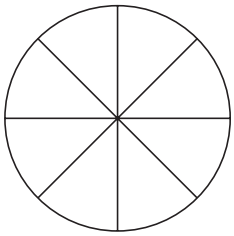


7. A square has an area of 36 m^2. Calculate its *perimeter.*

8. Calculate the *perimeter* of this shape, giving your answer correct to the nearest centimetre:

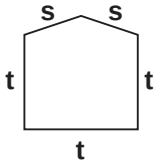


9. A circle of radius 32 cm is cut into 8 equal parts, as shown in the diagram.



Calculate the *perimeter* of each part, giving your answer correct to the nearest millimetre.

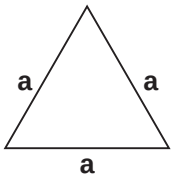
10. The total perimeter of a semicircle is 37 cm. Calculate the *radius* of the semicircle, giving your answer correct to the nearest millimetre.

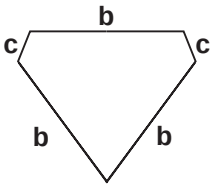
11. 

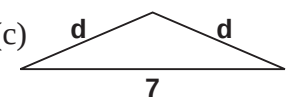
The perimeter of this shape is $3t + 2s$.

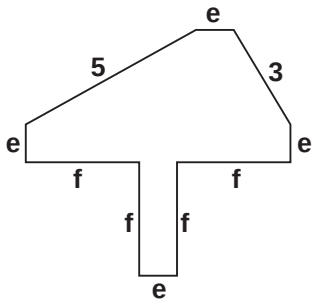
$\mathbf{p = 3t + 2s}$

Write an expression for the perimeters of each of these shapes.
Write each expression in its simplest form.

(a) 

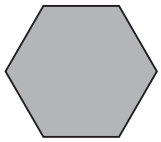
(b) 

(c) 

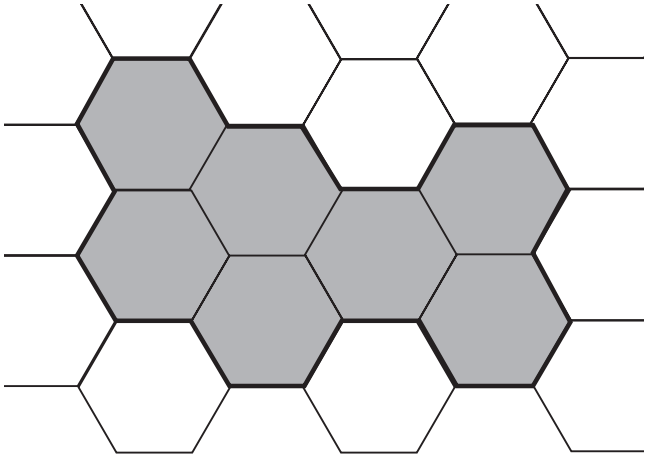
(d) 

(KS3/95/Ma/3-5/P1)

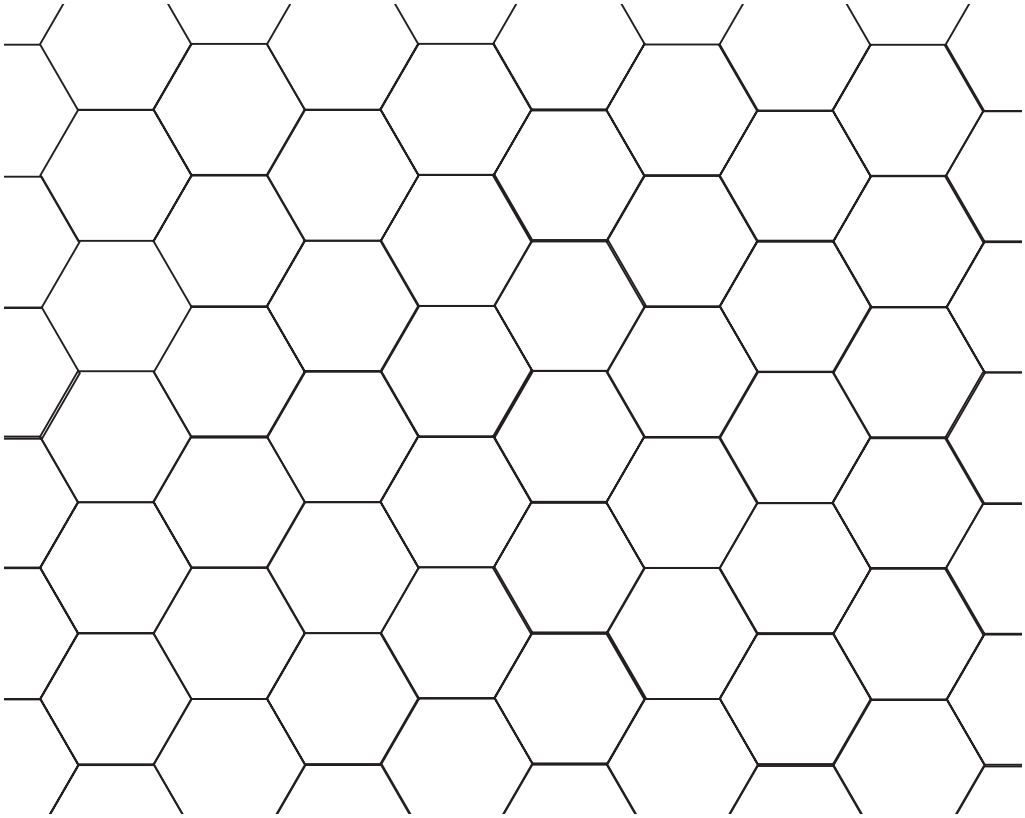
12. Each side of this hexagon is 1 cm long.



(a) The shaded shape below is made from 7 hexagon tiles.
Write down the perimeter of the shaded shape.



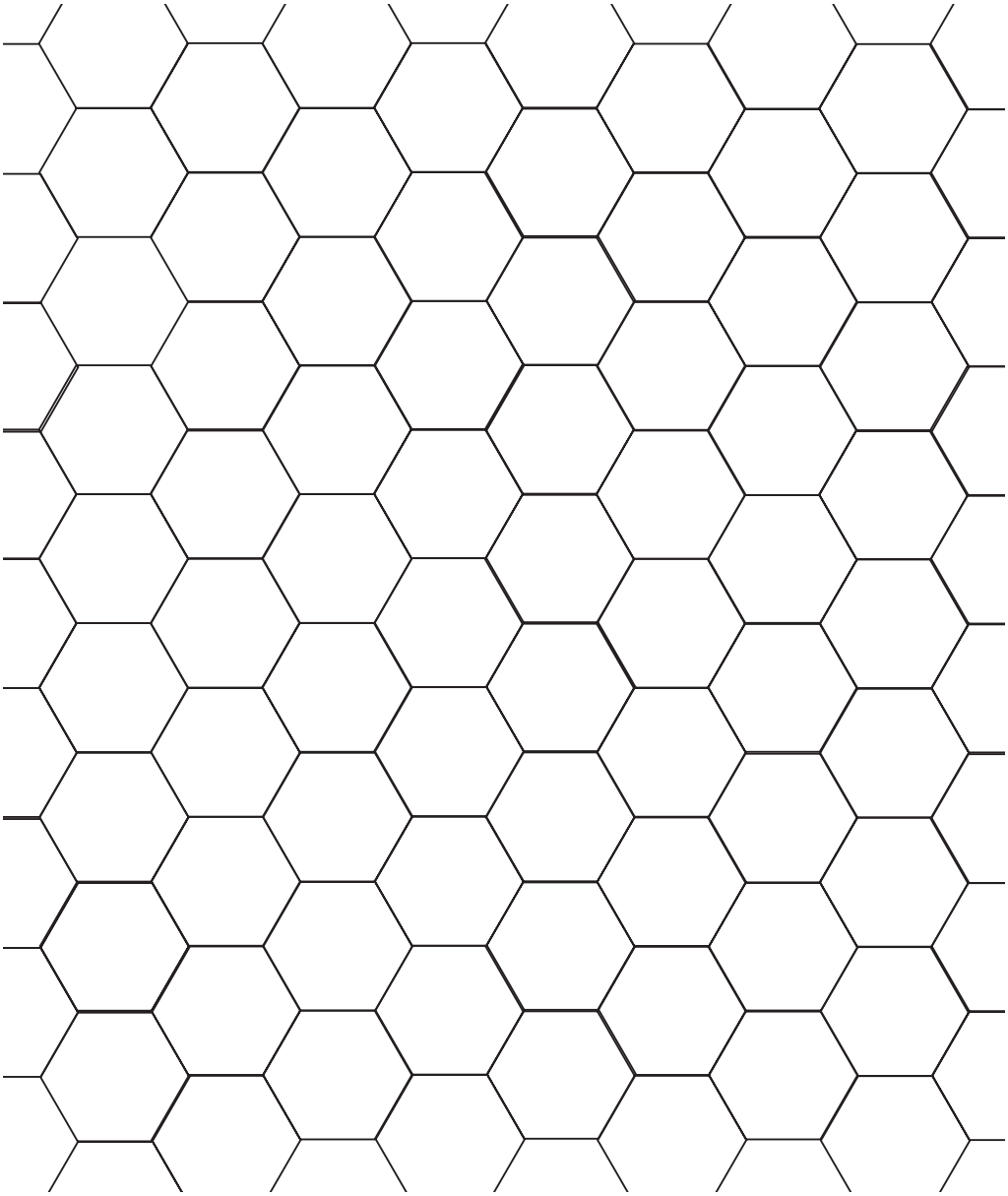
(b) On a copy of the following diagram, shade a shape made with 7 tiles which has a *smaller* perimeter.



(c) Explain what made its perimeter less than the perimeter of the first shape.

(d) On a copy of the following diagram, shade a shape made with 7 tiles which has the *biggest* possible perimeter.

(e) Explain what made your shape have the biggest possible perimeter.



(KS3/94/Ma/3-5/P2)

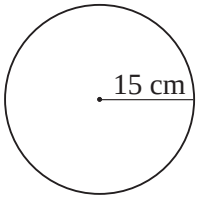
13. Wyn and Jay are using their wheelchairs to measure distances.

(a) The large wheel on Wyn's wheelchair has a diameter of 60 cm.
Wyn pushes the wheel round exactly once.
Calculate how far Wyn has moved.
Show your working.

(b) The large wheel on Jay's wheelchair has a diameter of 52 cm.
Jay moves her wheelchair forward 950 cm.
Calculate how many times the large wheel goes round.
Show your working.

(KS3/96/Ma/Tier 5-7/P2)

14. (a) A circle has a radius of 15 cm.
Calculate the *area* of the circle.
Show your working.



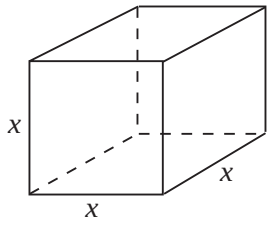
(b) A different circle has a *circumference* of 120 cm.
What is the radius of the circle?
Show your working.

(KS3/99/Ma/Tier 5-7/P2)

9.4 Surface Area and Volume of 3-D Shapes

In this section we calculate the volume and surface area of 3-D shapes such as *cubes, cuboids, prisms* and *cylinders.*

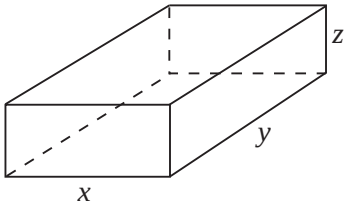
Cube



Volume $= x^3$

Surface area $= 6x^2$

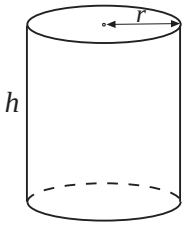
Cuboid



Volume $= xyz$

Surface area $= 2xy + 2xz + 2yz$

Cylinder



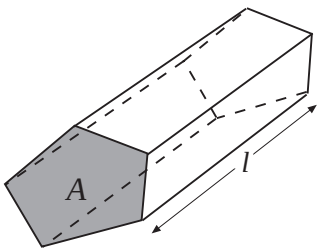
Volume $= \pi r^2 h$

Area of curved surface $= 2\pi rh$

Area of each end $= \pi r^2$

Total surface area $= 2\pi rh + 2\pi r^2$

Prism



A prism has a uniform cross-section

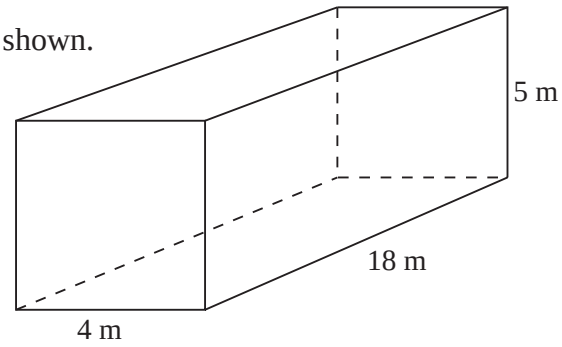
Volume = area of cross-section $\times$ length

$= Al$

Example 1

(a) Calculate the *volume* of the cuboid shown.

(b) Calculate the *surface area* of the cuboid shown.



Solution

(a) Volume $= 4 \times 18 \times 5$

$= 360 \text{ m}^3$

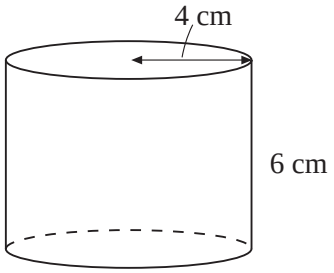
(b) Surface area $= (2 \times 4 \times 18) + (2 \times 4 \times 5) + (2 \times 5 \times 18)$

$= 144 + 40 + 180$

$= 364 \text{ m}^2$

Example 2

Calculate the *volume* and total *surface area* of the cylinder shown.



Solution

Volume $= \pi r^2 h = \pi \times 4^2 \times 6 = 96\pi$

$= 301.5928947 \text{ cm}^3$

$= 302 \text{ cm}^3$ (to 3 significant figures)

Area of curved surface $= 2\pi r h = 2 \times \pi \times 4 \times 6$

$= 48\pi$

$= 150.7964474 \text{ cm}^2$

Area of each end $= \pi r^2 = \pi \times 4^2$

$= 16\pi$

$= 50.26548246 \text{ cm}^2$

Total surface area $= 150.7964474 + (2 \times 50.26548246)$

$= 251.3274123 \text{ cm}^2$

$= 251 \text{ cm}^2$ (to 3 significant figures)

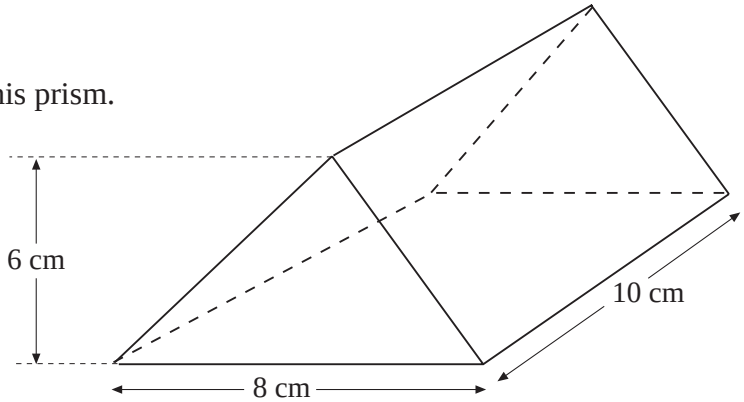
Note: From the working we can see that the area of the curved surface is 48π, and that the area of each end is 16π. The total surface area is therefore

$48\pi + (2 \times 16\pi) = 80\pi = 251.3274123 \text{ cm}^2$

$= 251 \text{ cm}^2$ (to 3 significant figures)

Example 3

Calculate the *volume* of this prism.



Solution

Area of end of prism $= \frac{1}{2} \times 8 \times 6$

$= 24 \text{ cm}^2$

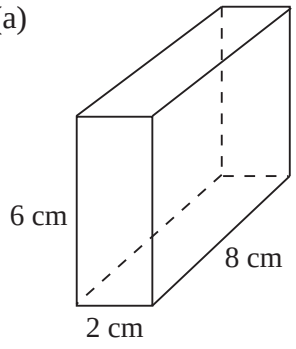
Volume of prism $= 24 \times 10$

$= 240 \text{ cm}^3$

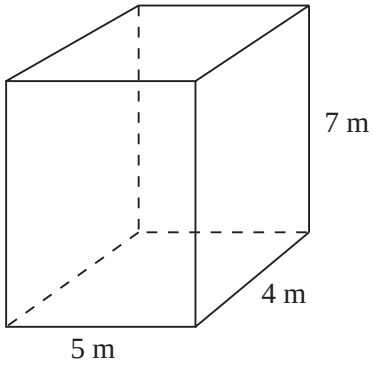
Exercises

1. Calculate the *volume* and *surface area* of each of the following cuboids:

(a)

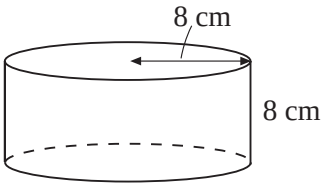


(b)

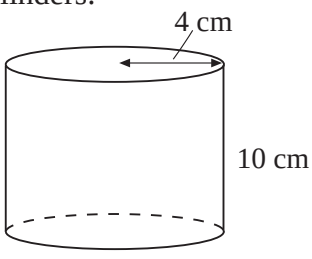


2. Giving your answers correct to 3 significant figures, calculate the *volume* and *total surface area* of each of the following cylinders:

(a)

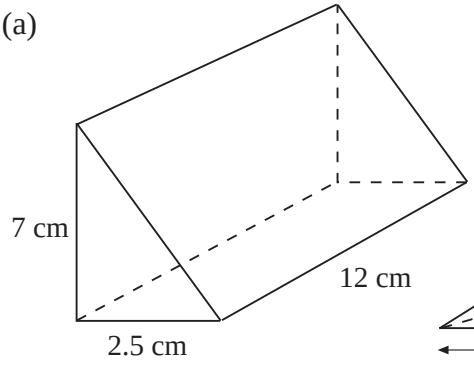


(b)

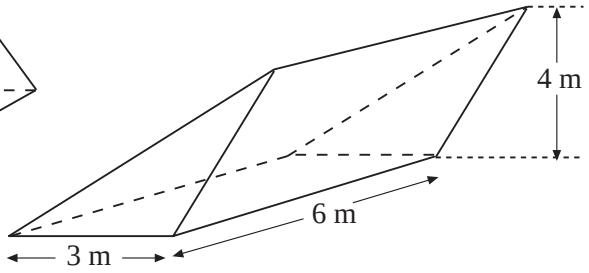


3. Calculate the *volume* of each of the following prisms:

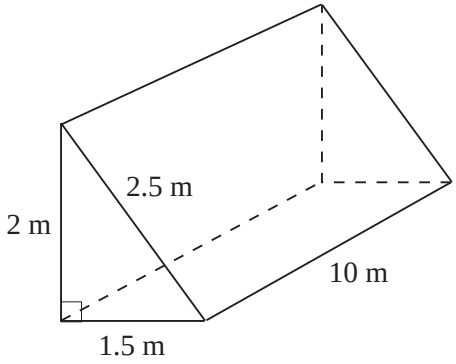
(a)



(b)

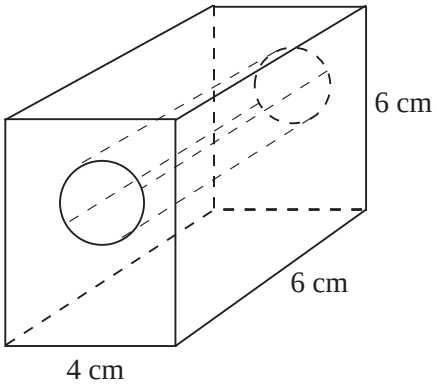


4. Calculate the *volume* and *surface area* of the following prism:



5. The diagram shows a wooden block that has had a hole drilled in it. The diameter of the hole is 2 cm.

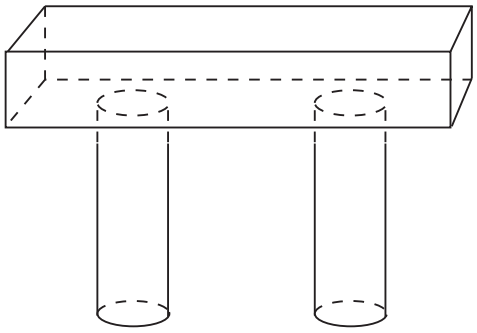
Calculate the *volume* of this solid, giving your answer correct to 2 decimal places.



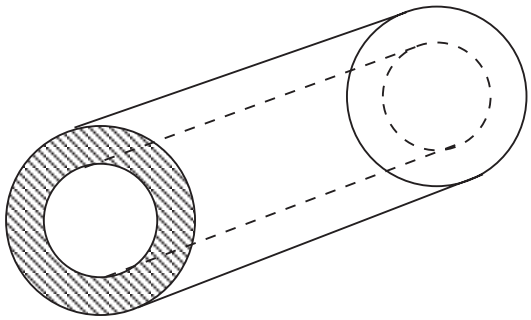
6. A concrete beam is to rest on two concrete pillars. The beam is a cuboid with sides of length 0.5 m, 3 m and 0.4 m.

The pillars have diameter 0.4 m and height 2 m.

Calculate the *total volume* of concrete needed to make the beam and the pillars. Round your answer to a sensible level of accuracy.



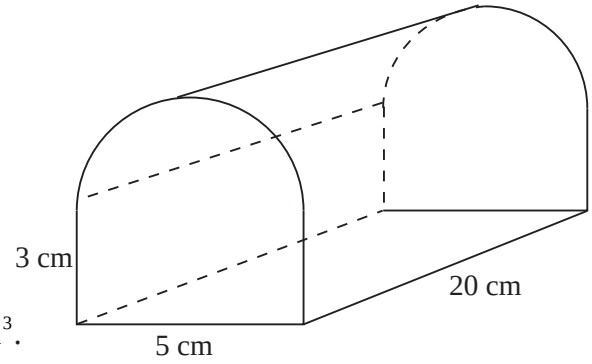
7. The diagram shows the cross-section of a pipe of length 50 cm.
The inner diameter of the pipe is 20 cm and the outer diameter is 30 cm.



(a) Calculate the *volume* of metal needed to make the pipe. Round your answer to a sensible level of accuracy.

(b) Calculate the *total surface area* of the pipe, including the inside surface. Round your answer to a sensible level of accuracy.

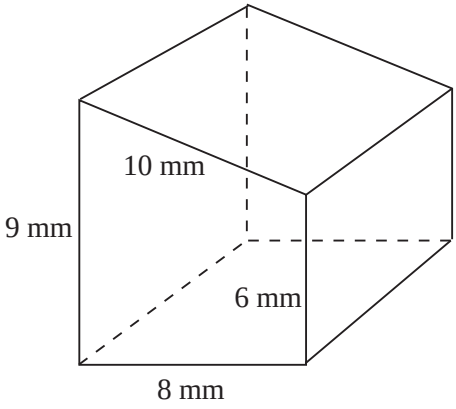
8. The diagram shows a prism.
The cross-section of the prism consists of a rectangle and a semicircle.



(a) Calculate the *volume* of the prism. Give your answer to the nearest cm^3.

(b) Calculate the *total surface area* of the prism. Give your answer to the nearest cm^2.

9. The volume of the prism shown is 720 mm^3.



(a) Determine the *length* of the prism.

(b) Calculate the *surface area* of the prism.

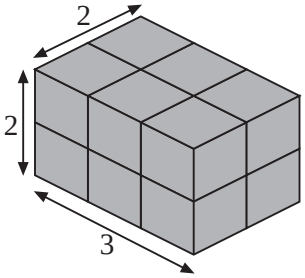
10. A cylinder has a diameter of 12 cm and a curved surface area of 132π or $415\ \text{cm}^2$ (to 3 significant figures).

(a) Determine the *height* of the cylinder.

(b) Calculate the *volume* of the cylinder, giving your answer to the nearest cm^3.

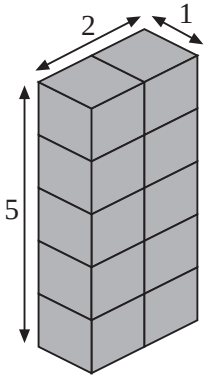
11. (a) These cuboids are made from small cubes.
Write *how many small cubes* there are in each cuboid.
The first is done for you.

(i)

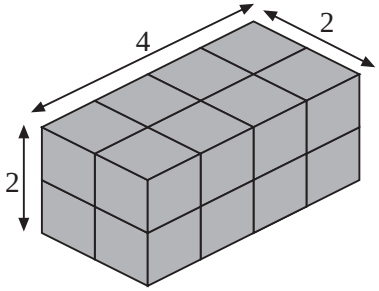


Cube (i) is made from 12 small cubes.

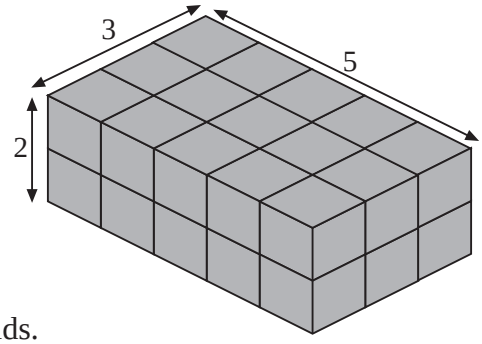
(ii)



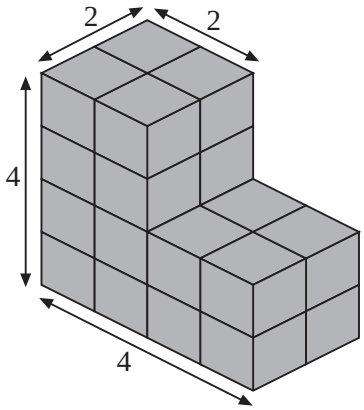
(iii)



(iv)

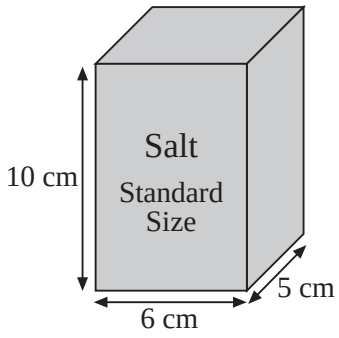


(b) This shape is made with two cuboids.
Write *how many cubes* there are in this shape.

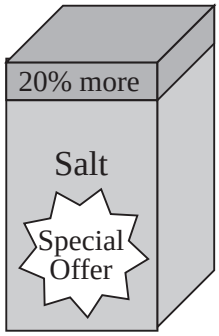


(KS3/98/Ma/Tier 3-5/P1)

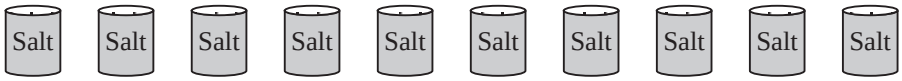
12. (a) What is the volume of this *standard size* box of salt?



(b) What is the volume of this *special offer* box of salt, which is 20% bigger?



The standard size box contains enough salt to fill up 10 salt pots.

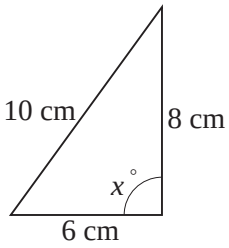


(c) How many salt pots may be filled up from the *special offer* box of salt?

(KS3/96/Ma/Tier 5-7/P2)

13. (a) Look at this triangle.

NOT TO SCALE

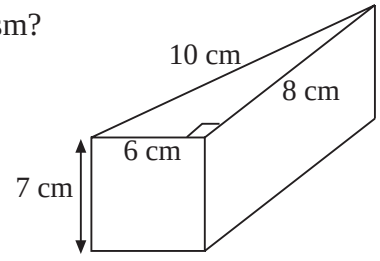


Show working to explain why angle *x* *must* be a right angle.

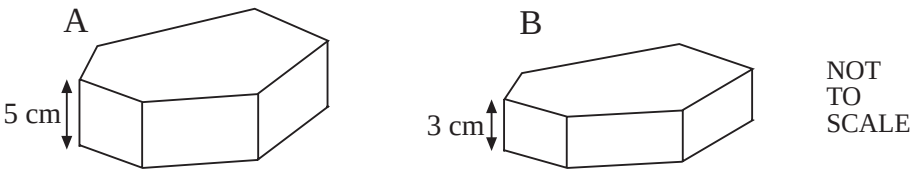
(b) What is the volume of this prism?

You *must* show *each step* in your working.

NOT TO SCALE



(c) Prisms A and B have the same cross-sectional area.

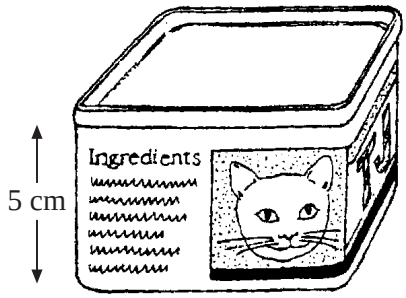


Copy and complete the table:

	Prism A	*Prism B*
height	5 cm	3 cm
volume	200 cm^3	..... cm^3

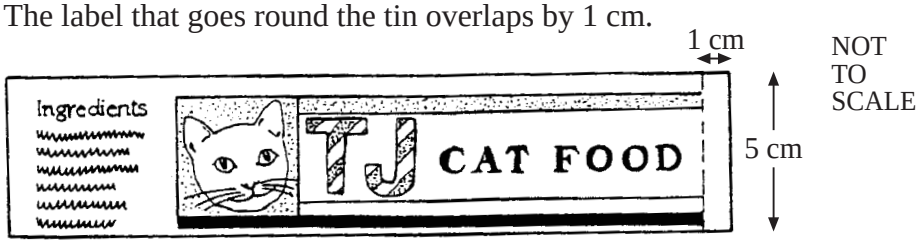
(KS3/99/Ma/Tier 5-7/P1)

14. TJ's Cat Food is sold in tins shaped like this.
Each tin has an internal height of 5 cm.



(a) The area of the lid of the tin is 35 cm^2.
Work out the volume of cat food that the tin contains.

(b) The label that goes round the tin overlaps by 1 cm.



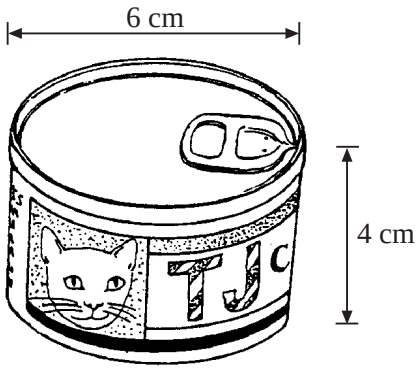
The area of the label is 134 cm^2.

Work out the distance around the tin.

Show your working.

TJ's Cat Food plans to use tins that are the shape of cylinders.

The internal measurements of a tin are shown.



(c) Work out the volume of cat food that the tin contains.
Show your working.

(KS3/95/Ma/Levels 5-7/P2)