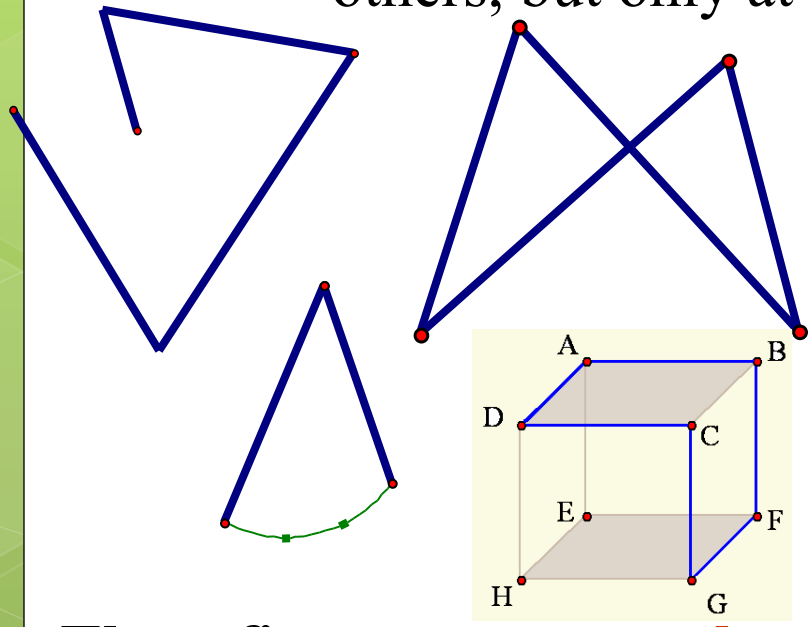
Lesson 3-4

Polygons

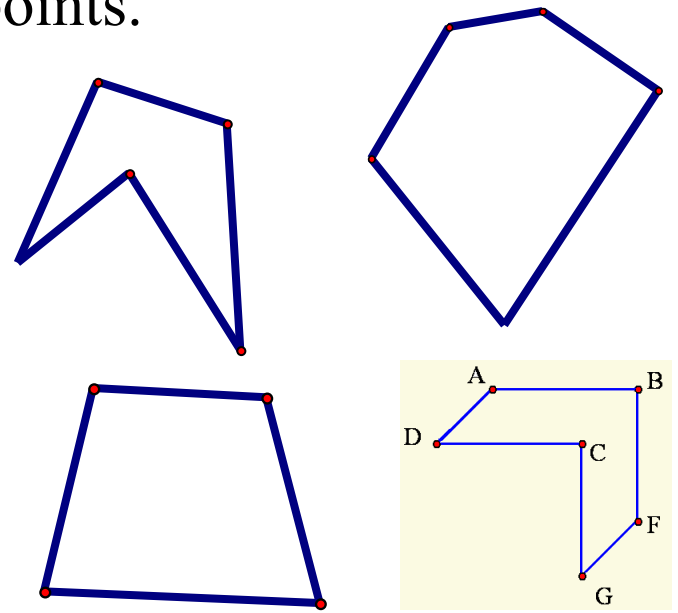
Modified by Lisa Palen

Polygons

Definition: A closed figure formed by a finite number of coplanar segments so that each segment intersects exactly two others, but only at their endpoints.



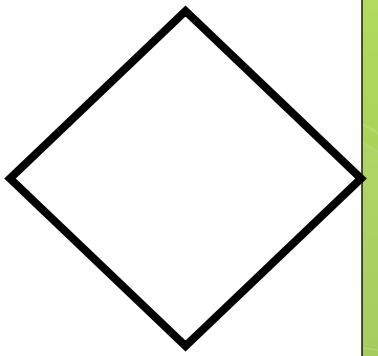
These figures are not polygons



These figures are polygons

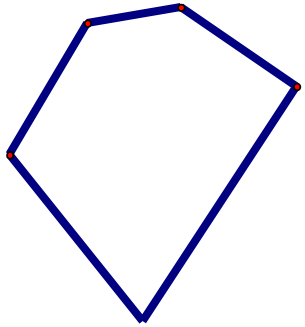
Classifications of a Polygon

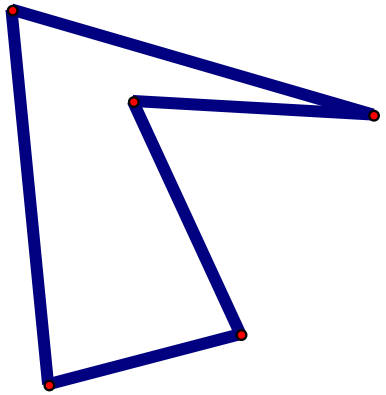
Regular: A polygon in which all angles are congruent and all sides are congruent



That's an equiangular equilateral polygon.

Irregular: A polygon that is not regular



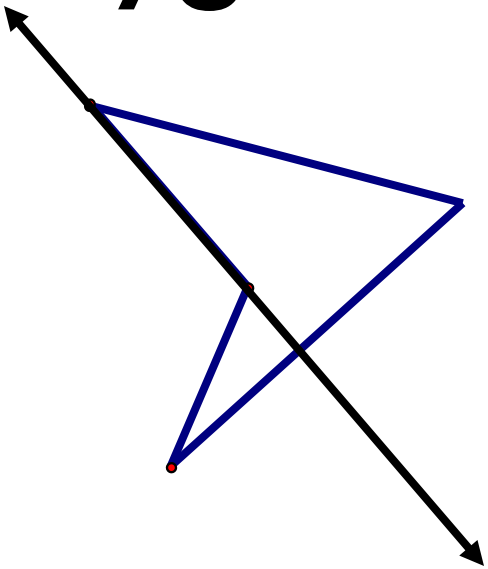


Classifications of a Polygon

Concave: Like a cave.
Has indentations.

A polygon that has at least one angle that measures **more than** 180° (a reflex angle)

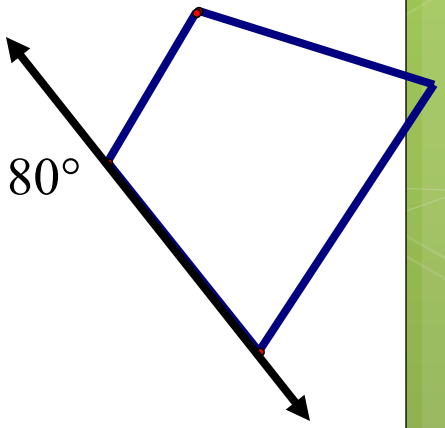
A polygon for which there is a line containing a side of the polygon and a point in the interior of the polygon.



Convex: Has no indentations.

A polygon with **no** angles measuring more than 180°

No line containing a side of the polygon contains a point in its interior

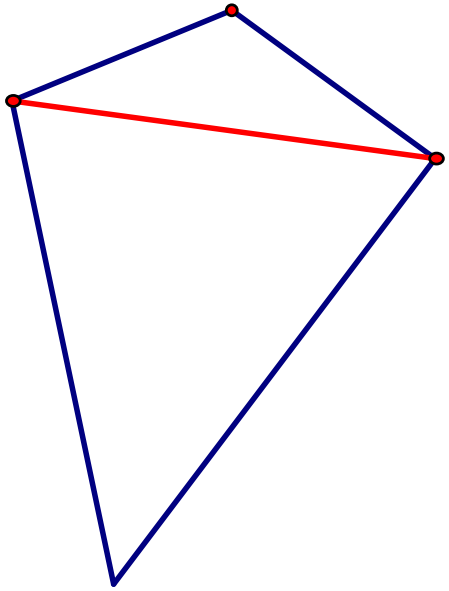


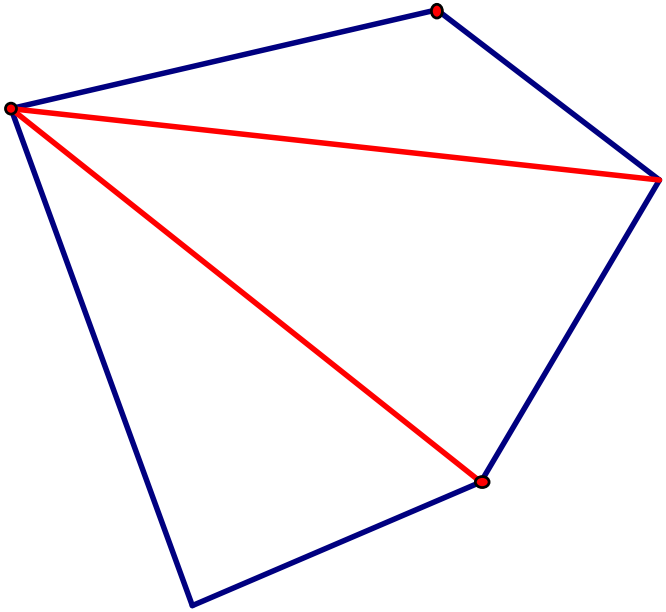
Polygon Names

3 sides	→	Triangle
4 sides	→	Quadrilateral
5 sides	→	Pentagon
6 sides	→	Hexagon
7 sides	→	Heptagon
8 sides	→	Octagon
9 sides	→	Nonagon
10 sides	→	Decagon
12 sides	→	Dodecagon
n sides	→	n-gon

Diagonal of a Polygon

a segment connecting nonconsecutive vertices of a polygon



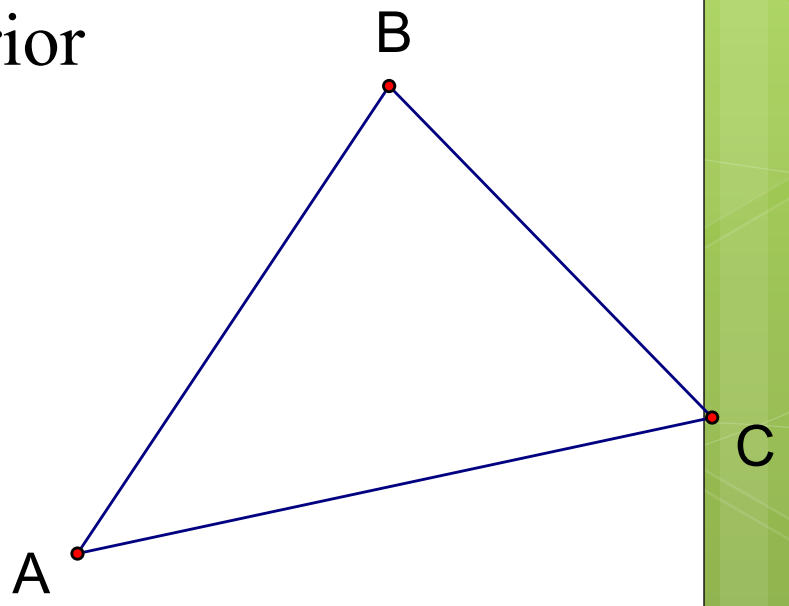


Interior Angle of a Polygon

The **interior angles** of a polygon are the angles inside the polygon, formed by two adjacent sides.

For example, ΔABC has interior angles:

□ ABC, □ BAC, □ BCA

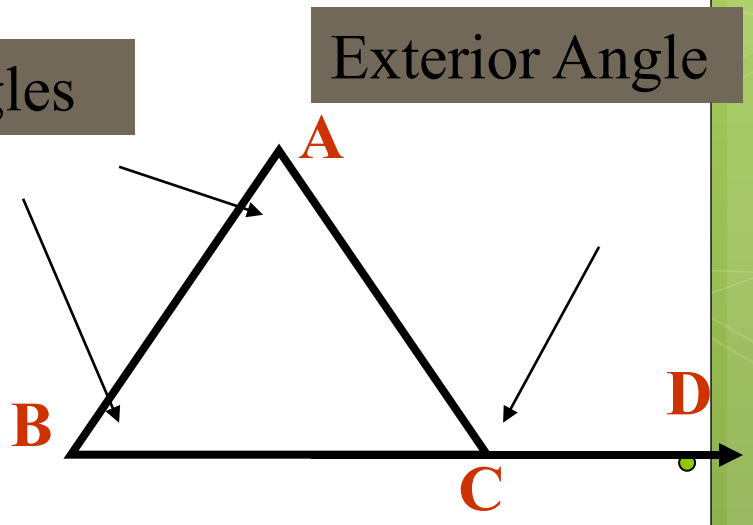


Exterior Angle of a Polygon

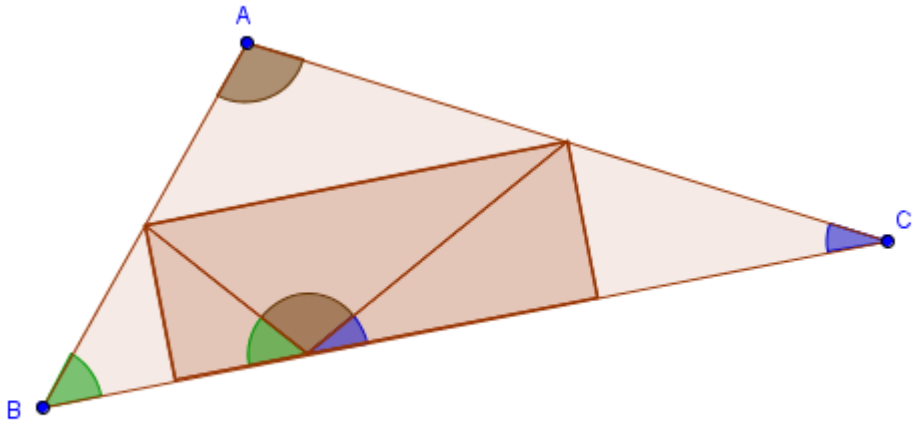
An **exterior angle** of a polygon is an angle that forms a linear pair with an interior angle. It is an angle outside the polygon formed by one side and one extended side of the polygon.

For example, ΔABC has exterior angle:

□ ACD. It forms a linear pair with □ ACB.



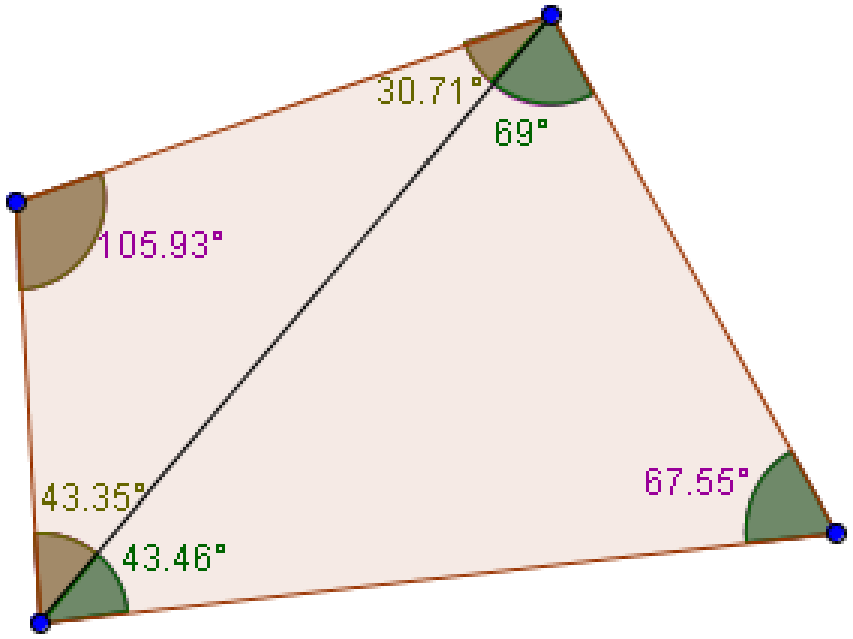
What is the sum of the measures of the interior angles of a 3-gon?



m<A + m<B + m<C = 180□

GeoGebra Applet at Geometry Online

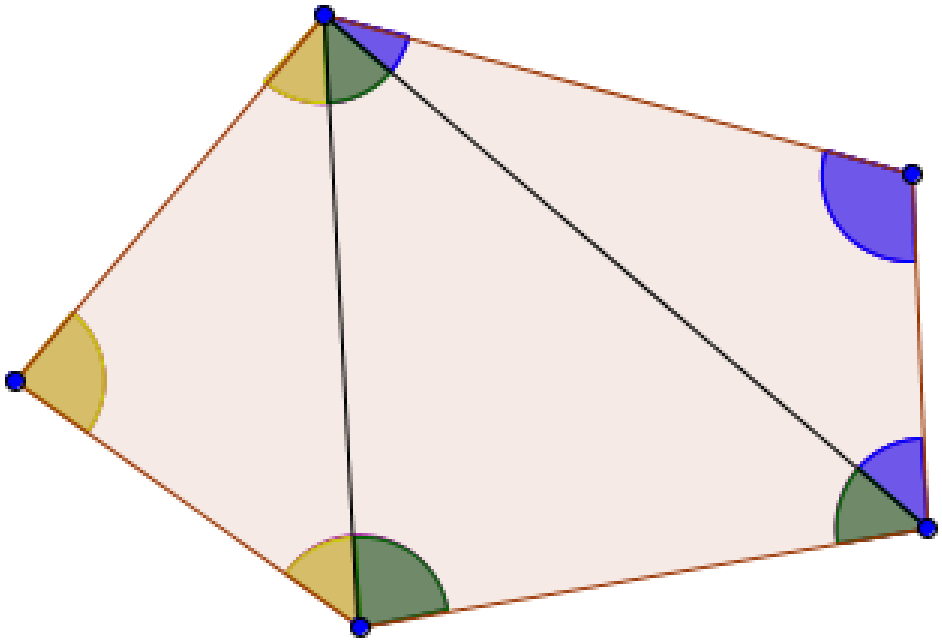
What is the sum of the measures of the interior angles of a convex 4-gon?



180□+ 180□= 360□

GeoGebra Applet at Geometry Online

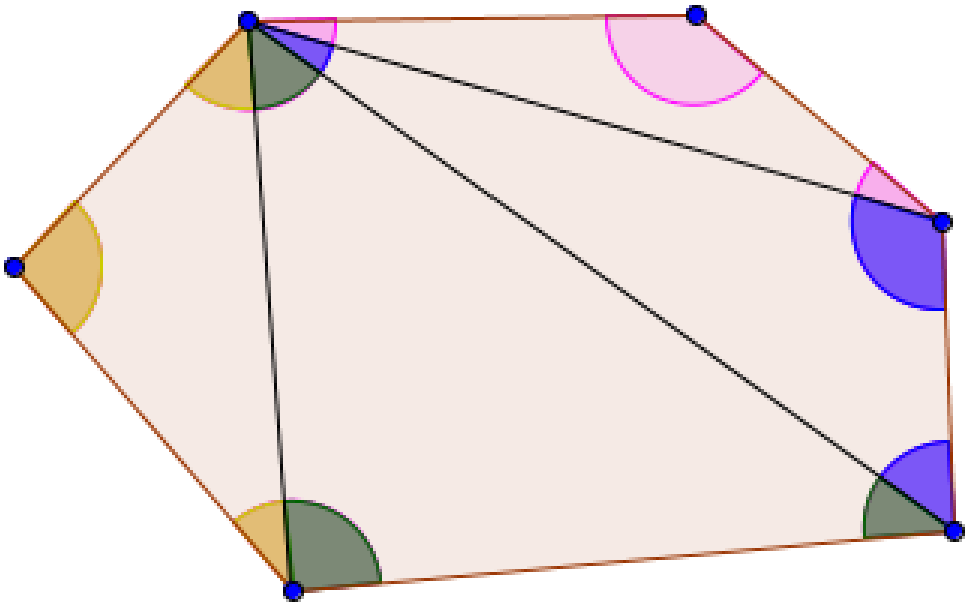
What is the sum of the measures of the interior angles of a convex 5-gon?



$180° + 180° + 180° = 540°$

GeoGebra Applet at Geometry Online

What is the sum of the measures of the interior angles of a convex 6-gon? (a hexagon)



$180° + 180° + 180° + 180° = 720°$

GeoGebra Applet at Geometry Online

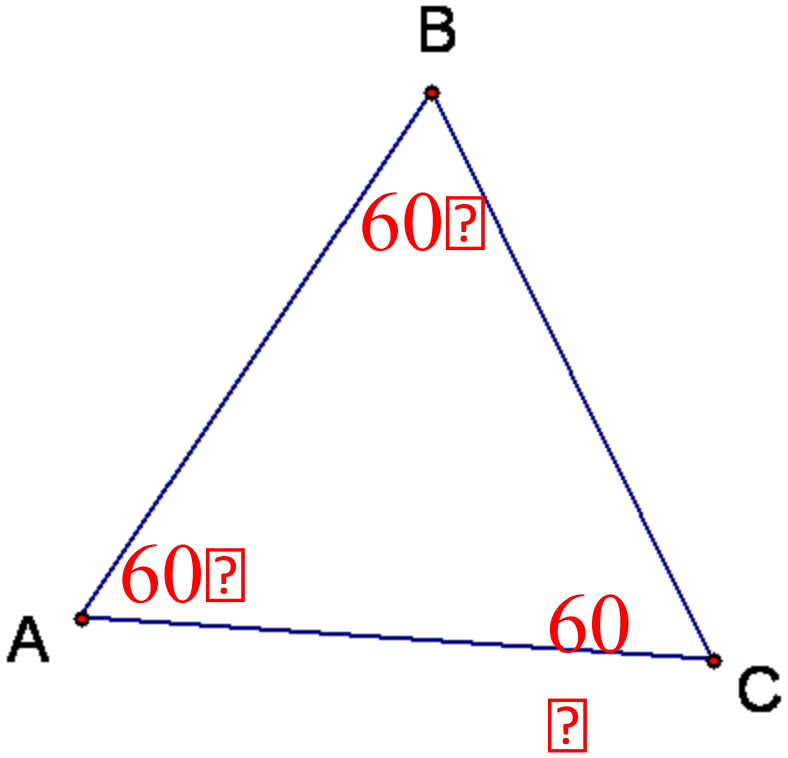
What is the sum of the measures of the interior angles of a regular n-gon?

$$\text{sum} = (n-2)180$$

What is the sum of the measures of the interior angles of a convex n-gon?

Polygon	Number of Sides	Sum of Measures of Interior Angles
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°
n-gon	n	(n - 2)180°

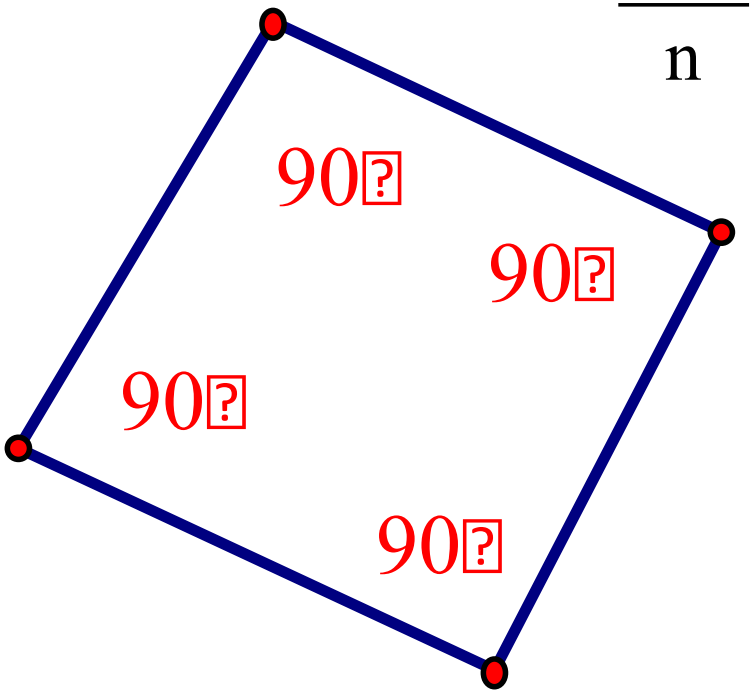
What is the measure of each interior angle of a regular 3-gon?



$$\frac{\text{sum}}{n} = \frac{180°}{3} = 60°$$

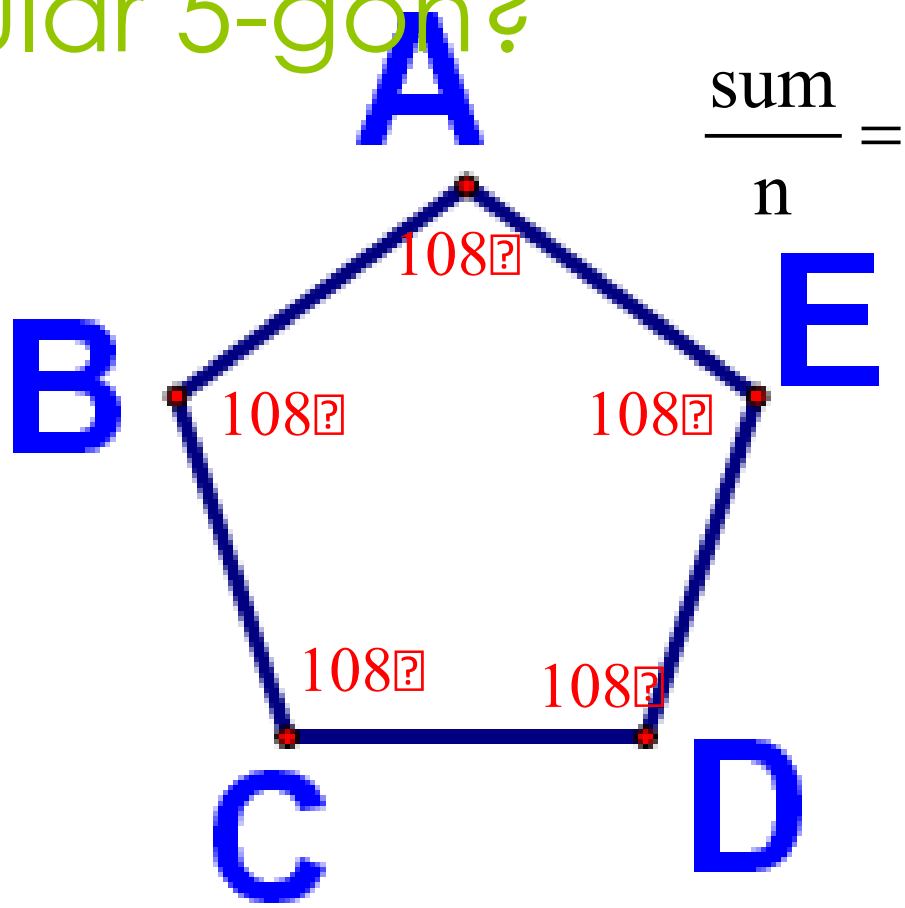
What is the measure of each interior angle of a regular 4-gon?

$$\frac{\text{sum}}{n} = \frac{360°}{4} = 90°$$



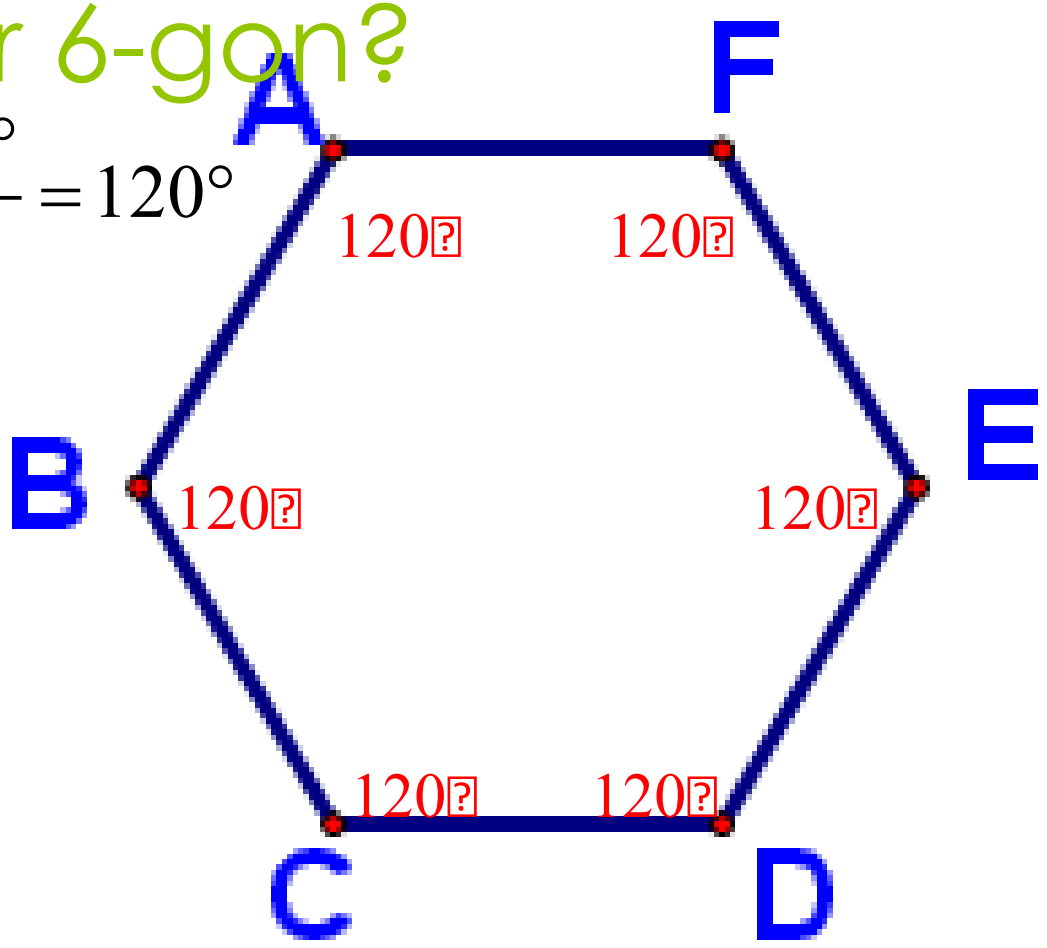
What is the measure of each interior angle of a regular 5-gon?

$$\frac{\text{sum}}{n} = \frac{540°}{5} = 108°$$



What is the measure of each interior angle of a regular 6-gon?

$$\frac{\text{sum}}{n} = \frac{720°}{6} = 120°$$



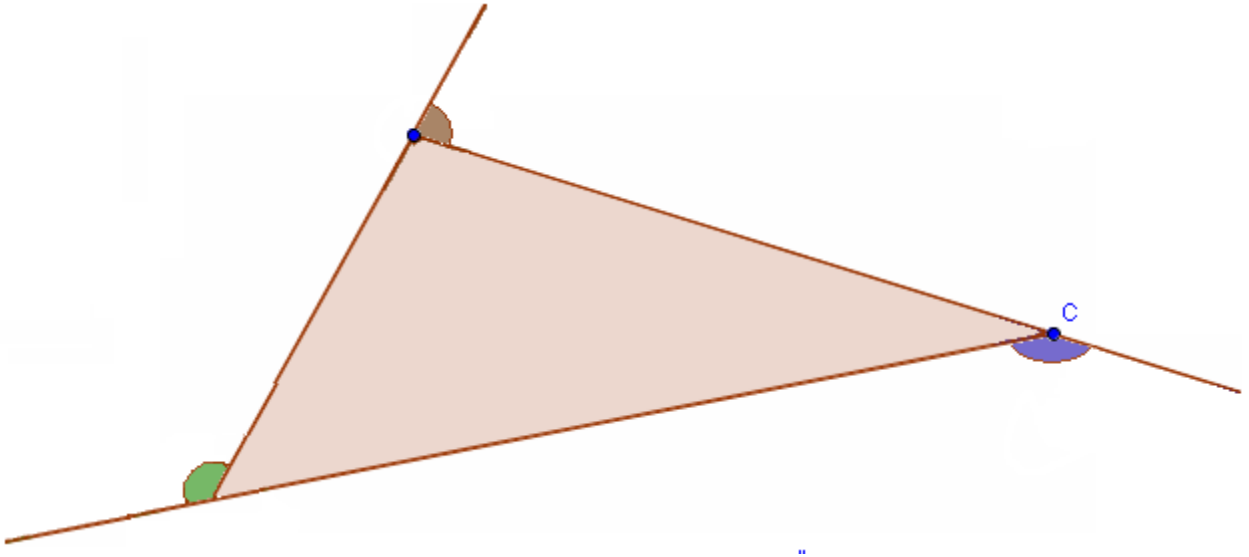
What is the measure of each interior angle of a regular n-gon?

$$\frac{\text{sum}}{n} = \frac{(n-2)180}{n}$$

What is the measure of each interior angle of a regular n-gon?

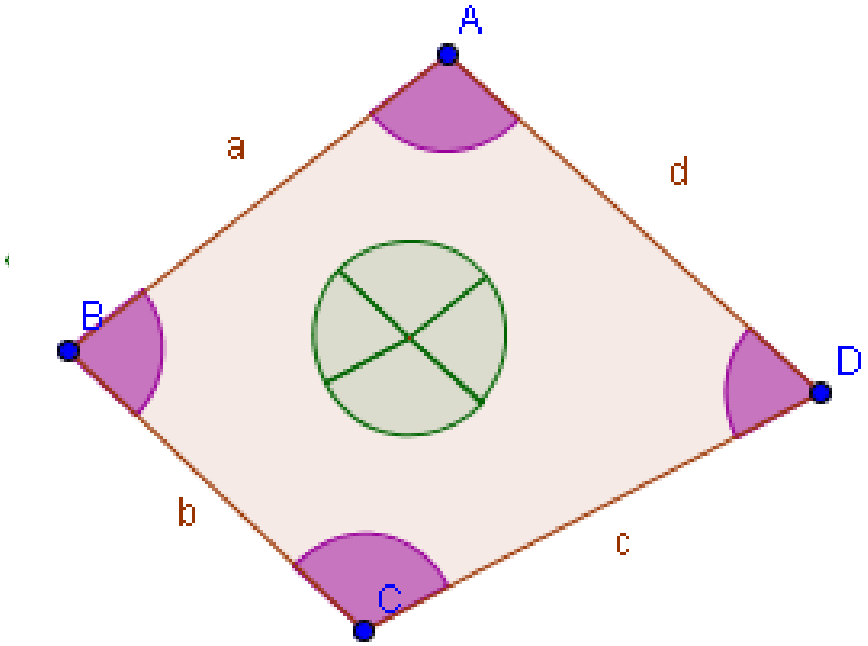
Polygon	Number of Sides	Measure of Each Interior Angle
Triangle	3	60°
Quadrilateral	4	90°
Pentagon	5	108°
Hexagon	6	120°
n-gon	n	(n - 2)180°/n

What is the sum of the measures of the exterior angles of a 3-gon? (a triangle)



Sum of Measures of Exterior Angles = 360□

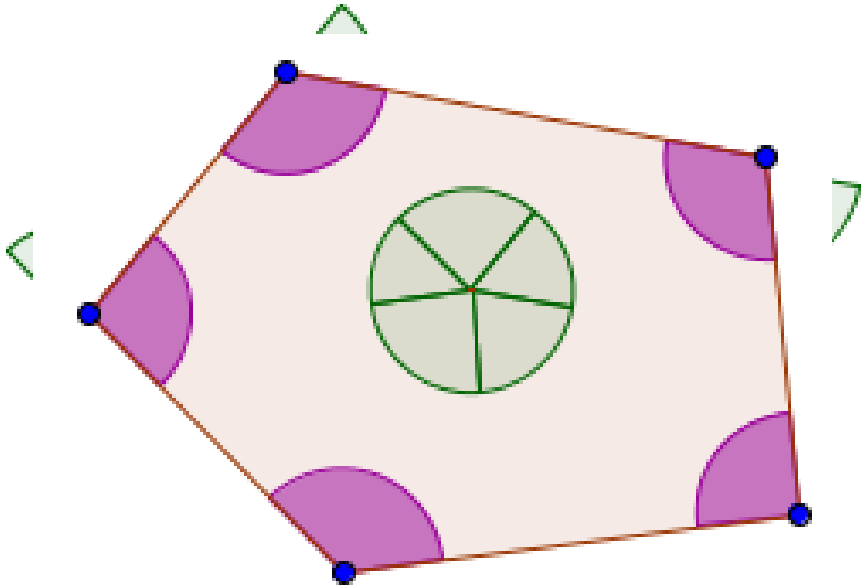
What is the sum of the measures of the exterior angles of a 4-gon? (a quadrilateral)



Sum of Measures of Green Angles = 360°

GeoGebra Applet at Geometry Online

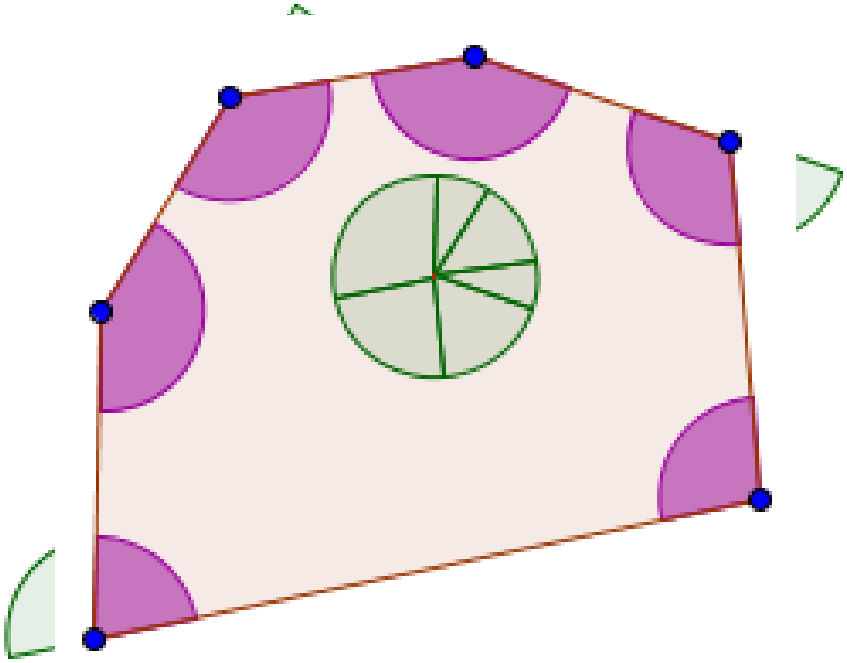
What is the sum of the measures of the exterior angles of a 5-gon? (a pentagon)



Sum of Measures of Green Angles = 360□

GeoGebra Applet at Geometry Online

What is the sum of the measures of the exterior angles of a 6-gon? (a hexagon)



Sum of Measures of Green Angles = 360□

GeoGebra Applet at Geometry Online

What is the sum of the measures of the exterior angles of a convex n-gon?

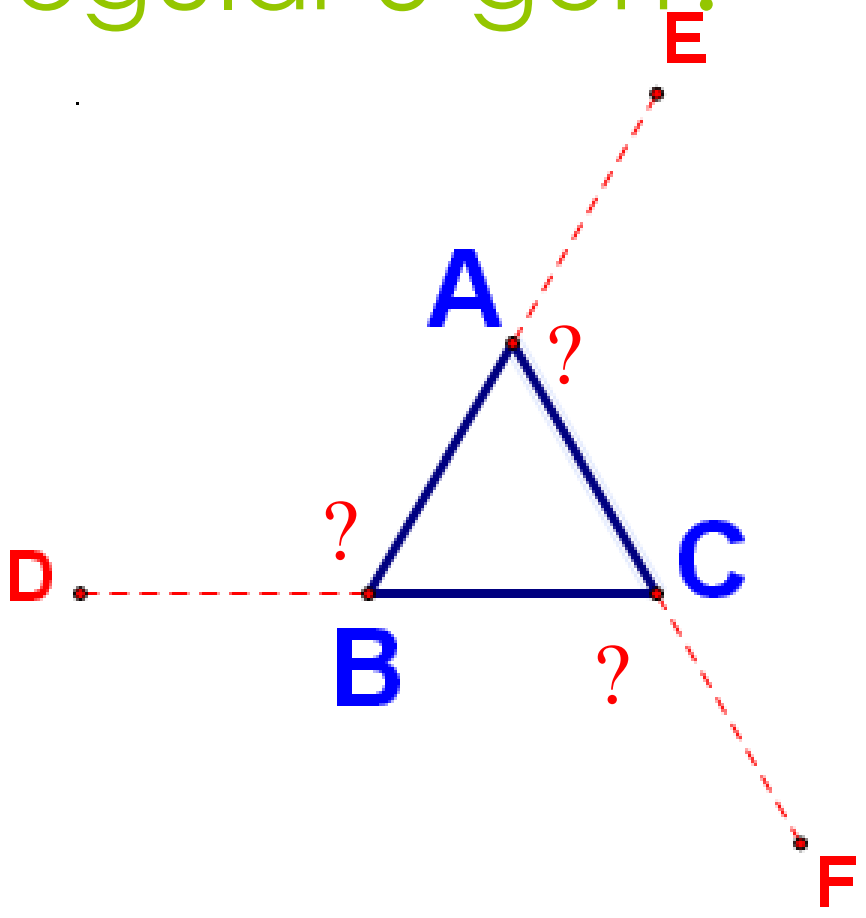
$$\text{sum} = 360°$$

Yes, always, no matter what n is!

What is the sum of the measures of the exterior angles of a convex n-gon?

Polygon	Number of Sides	Sum of Measures of Exterior Angles
Triangle	3	360°
Quadrilateral	4	360°
Pentagon	5	360°
Hexagon	6	360°
n-gon	n	360°

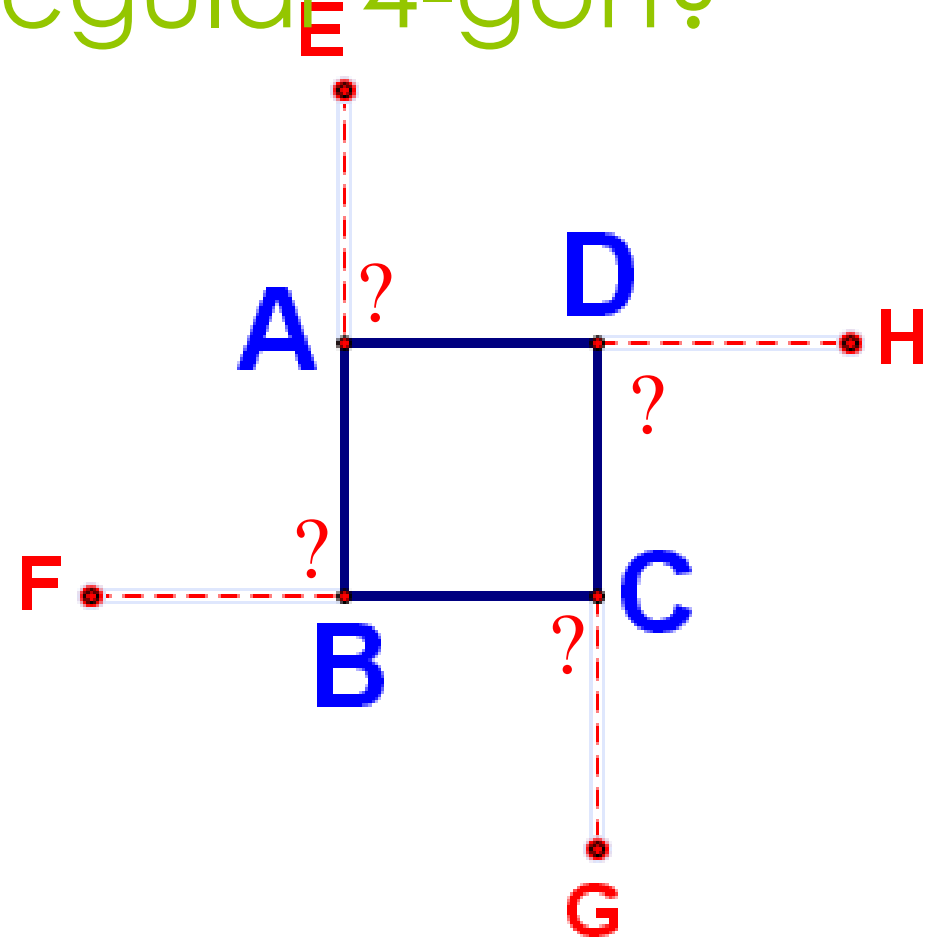
What is the measure of each exterior angle of a regular 3-gon?



$$\frac{\text{sum}}{n} = \frac{360°}{n}$$

$$= \frac{360°}{3} = 120°$$

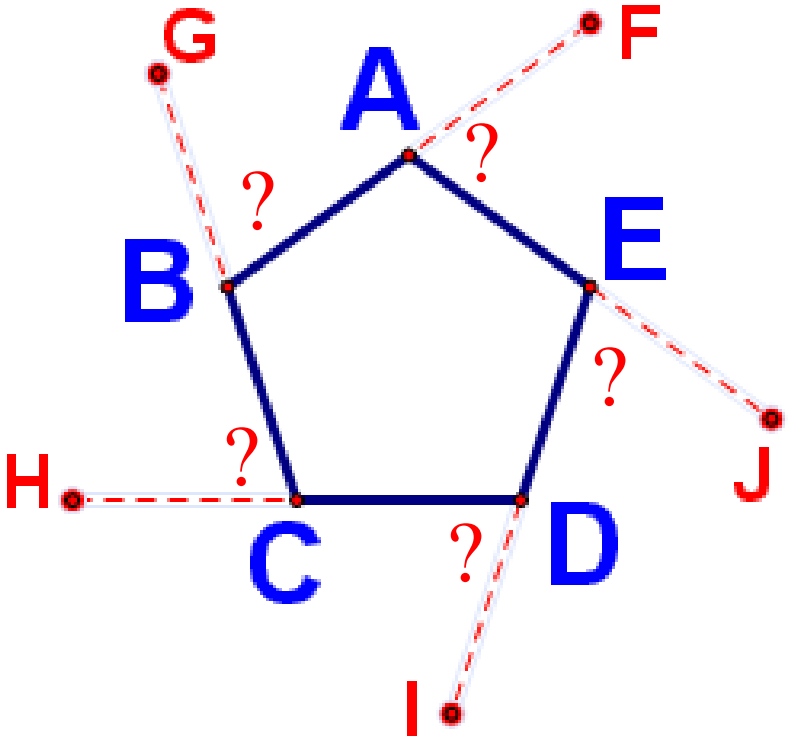
What is the measure of each exterior angle of a regular 4-gon?



$$\frac{\text{sum}}{n} = \frac{360°}{n}$$

$$= \frac{360°}{4} = 90°$$

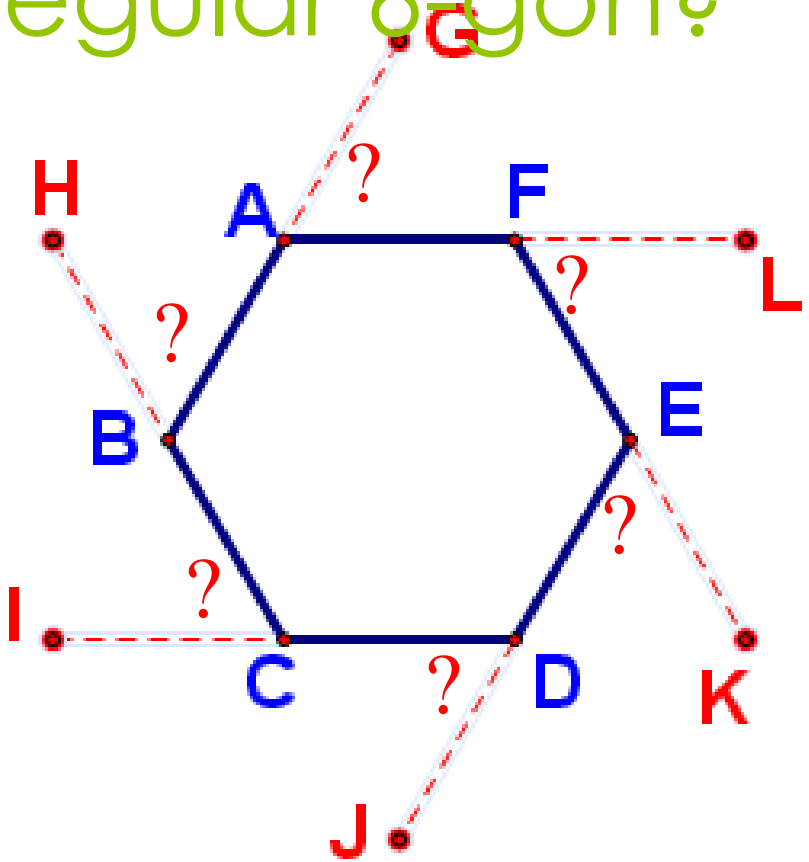
What is the measure of each exterior angle of a regular 5-gon?



$$\frac{\text{sum}}{n} = \frac{360°}{n}$$

$$= \frac{360°}{5} = 72°$$

What is the measure of each exterior angle of a regular 6-gon?



$$\frac{\text{sum}}{n} = \frac{360°}{n}$$

$$= \frac{360°}{6} = 60°$$

What is the measure of each exterior angle of a regular n-gon?

$$\frac{\text{sum}}{n} = \frac{360}{n}$$

What is the measure of each exterior angle of a regular n-gon?

Polygon	Number of Sides	Measure of Each Exterior Angle
Triangle	3	120°
Quadrilateral	4	90°
Pentagon	5	72°
Hexagon	6	60°
n-gon	n	360°/n

For a *convex polygon* with n sides:

The sum of the measures of the interior angles is

$$(n-2)\cdot 180$$

The sum of the measures of the exterior angles is

$$360°$$

For a *regular polygon* with n sides:

The measure of one interior angle is

$$\frac{(n-2)\cdot 180}{n}$$

The measure of one exterior angle is

$$\frac{360}{n}$$