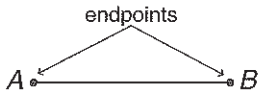
Geometry

Naming Points, Lines, Line Segments, and Rays

Geometry is the branch of mathematics that deals with lines, points, curves, angles, surfaces, and solids, and with relationships among these things. The chart below contains basic terms used in geometry.

Term	Definition	Symbol
point	a location on an object or a position in space	A point is shown by a capital letter.
line	a connected set of points that extends without end in two directions	Two letters on the line are used to name the line. Line PQ is written $\overleftrightarrow{PQ}$. This line can also be called $\overleftrightarrow{QP}$.
line segment	two points (endpoints) and the straight path between them	 A line segment is a section of a line. It is named by its endpoints. Line segment AB is written $\overline{AB}$. This line segment can also be called $\overline{BA}$.
ray	part of a line that extends in one direction	Ray WX is written $\overrightarrow{WX}$. The endpoint is always given first.

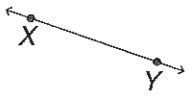
PRACTICE

Name each figure.

1. _____

2. _____

3. _____

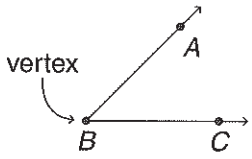
4.  _____

5. _____

6. _____

Naming Angles

An **angle** is formed by two rays that share the same endpoint. The endpoint is the **vertex** of the angle, and the two rays form the **sides** of the angle. The symbol for an angle is ∠.

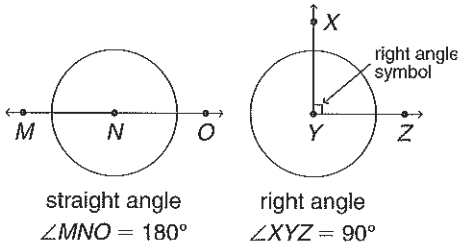


- Angles can be named by their vertex letter, or with three letters. The angle at right is ∠*B*, or ∠*ABC*, or ∠*CBA*. If two or more angles share the same vertex, use three letters; the vertex letter must be the middle letter.

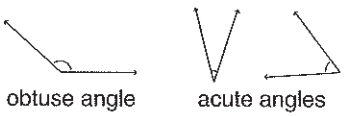
An angle is measured in by the number of degrees between the rays. Think of an angle as part of a circle with its vertex at the center. A circle is 360°.

- A **straight angle** cuts the circle in half. It has 180°.
- A **right angle** is $\frac{1}{4}$ of a circle. It is 90°.

  The symbol for a right angle is a small square in the angle.

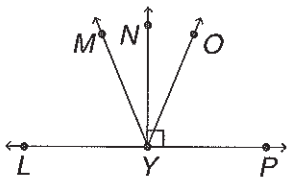


- An **acute angle** is less than 90°.
- An **obtuse angle** is greater than 90°, but less than 180°.



PRACTICE

Use the drawing below to answer Questions 1–4. Circle the correct answer.



1. Which of these does *not* name an angle in the above drawing?

   A ∠ *MYN* **C** ∠ *YPO*
   B ∠ *NYL* **D** ∠ *OYL*

2. Which is an obtuse angle?

   F ∠ *MYP* **H** ∠ *MYO*
   G ∠ *OYP* **J** ∠ *NYP*

3. Which is an acute angle?

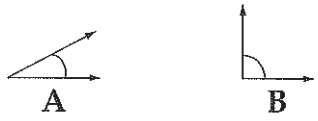
   A ∠ *LYO* **C** ∠ *NYP*
   B ∠ *PYM* **D** ∠ *LYM*

4. Which is a straight angle?

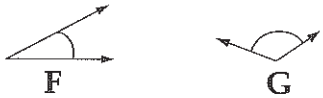
   F ∠ *OYP* **H** ∠ *LYP*
   G ∠ *MYO* **J** ∠ *OYL*

Circle the letter of the smaller angle in each pair.

5.



6.



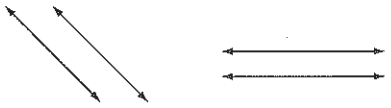
Investigating Lines

Lines that cross, or that will cross, are called **intersecting lines**. The point at which they cross is called the **point of intersection**.

Intersecting Lines

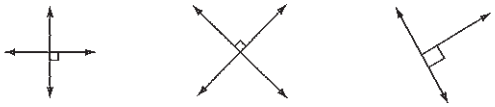
Lines that are always the same distance apart are **parallel lines**. Parallel lines will never intersect. The symbol || means "is parallel to."

Parallel Lines



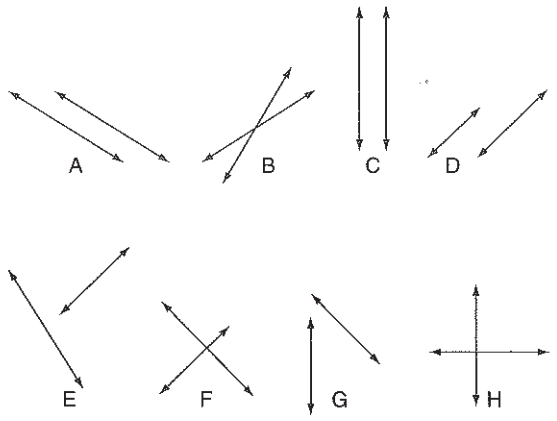
Lines that form right angles when they intersect are called **perpendicular lines**. The symbol ⊥ means "is perpendicular to."

Perpendicular Lines



PRACTICE

Use the figures below to answer Questions 1–3.

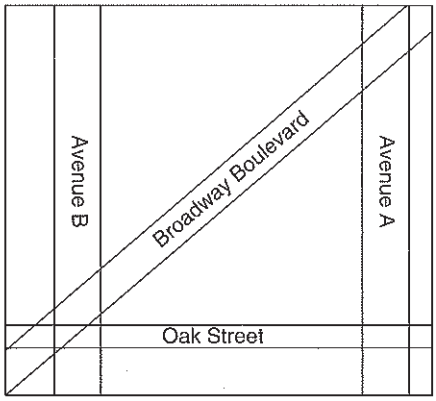


1. Which letters label pairs of lines that intersect? _____

2. Which letters label pairs of lines that are parallel? _____

3. Which letters label pairs of perpendicular lines? _____

Use this map for Questions 4 and 5.



4. Which streets are perpendicular to each other?

_____

_____

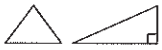
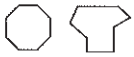
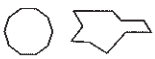
5. Which two streets are parallel?

_____

_____

Reviewing Polygons

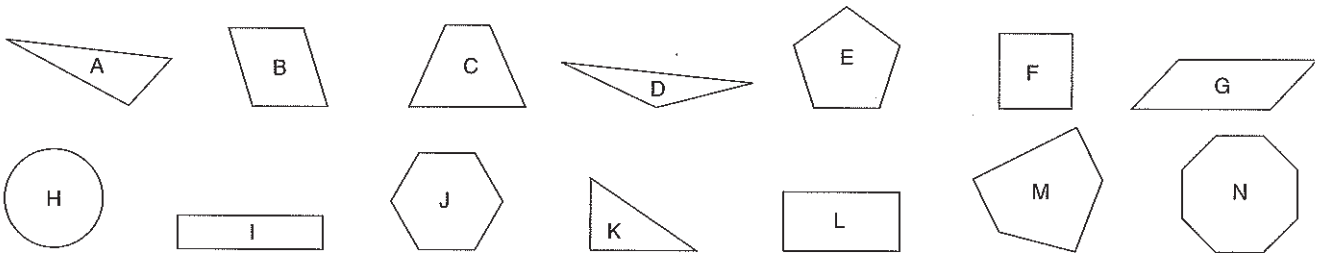
A **polygon** is a closed flat shape with three or more straight sides. The number of angles in a polygon is equal to the number of sides. Polygons are named according to the number of sides they have. Some polygons are listed below.

Name of Polygon	Example	Number of Sides/Angles
triangle		3
quadrilateral		4
pentagon		5
hexagon		6
octagon		8
decagon		10

A **regular polygon** is one in which all sides are the same length and all angles are equal. A square is a regular quadrilateral—it has four equal sides and four right angles (90°).

PRACTICE

Use this group of figures to answer the questions on this page.



1. Which three figures are triangles?

   _____

2. Which six figures are quadrilaterals?

   _____

3. Which two figures are pentagons?

   _____

4. Which figure is a hexagon?

   _____

5. Which figure is a square?

   _____

6. Which four polygons are regular?

   _____

7. Which figure is *not* a polygon?

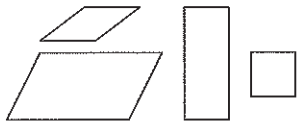
   _____

8. Which figure is an octagon?

   _____

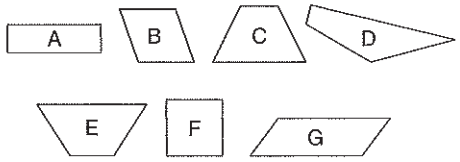
Recognizing Special Quadrilaterals

A quadrilateral is a polygon that has four sides and four angles. Within the family of quadrilaterals, figures are further classified according to their sides and angles.

Quadrilateral	Definition	Examples
parallelogram	A quadrilateral whose opposite sides are parallel. Opposite sides and opposite angles are equal.	
rectangle	A parallelogram with 4 right angles. Opposite sides are equal, but all 4 sides do not have to be equal.	
square	A rectangle with 4 equal sides	
rhombus	A parallelogram with 4 equal sides. Opposite angles are equal, but all 4 angles of a rhombus do not have to be equal. A square is a rhombus with 4 equal angles.	
trapezoid	A quadrilateral that has only 1 pair of parallel sides	

PRACTICE

Use this group of figures to answer Questions 1–4.



1. Write the letter of each figure that is a parallelogram. _____
2. Write the letter of each figure that is a rectangle. _____
3. Write the letter of each figure that is a trapezoid. _____
4. Write the letter of each figure that is a rhombus. _____

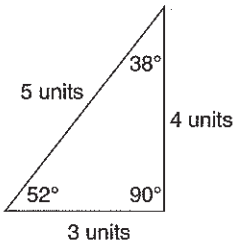
Write *True* or *False*.

5. A square is a parallelogram. _____
6. Some rectangles are squares. _____
7. A trapezoid is *not* a parallelogram. _____
8. All squares are rectangles. _____
9. All rectangles are parallelograms. _____
10. Some rhombuses are squares. _____

Working with Triangles

Here are two important facts about triangles:

- The sum of the three angles for every triangle is 180°.
- The sum of any two sides of a triangle must be greater than the third side.



$$\begin{array}{r} 90° \\ 38° \\ +52° \\ \hline 180° \end{array}$$

$3 + 4 > 5$
$3 + 5 > 4$
$4 + 5 > 3$

- Triangles can be named in two ways.

By lengths of sides:
equilateral—three equal sides*
isosceles—at least two equal sides
scalene—three sides of different lengths

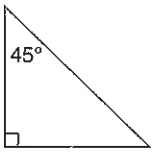
By measures of angles:
acute—each angle measures less than 90°
right—one right angle (90°)
obtuse—one angle measuring greater than 90°

The triangle shown above is both right and scalene.

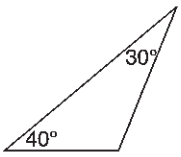
**Note*: An equilateral triangle is a regular polygon. It has three equal sides and three equal angles. The marks on each side indicate that the sides are congruent.

PRACTICE

For Questions 1–4, find the measure of the third angle. Then identify each triangle as acute, right, or obtuse.

1. 

_____

2. 

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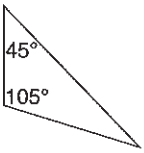
3. 

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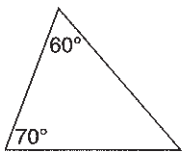
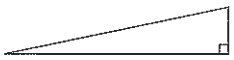
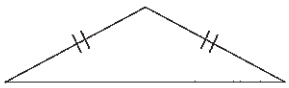
4. 

_____

Name each triangle by angle measures and by lengths of sides.

5. 
_____

6.
_____

7. 
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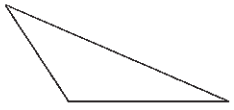
8. 
_____

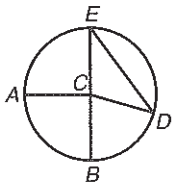
Investigating Circles

A **circle** is the set of all points that are the same distance from a given point called the **center**. When you see a dot at the center of the circle, you can assume it represents the center point of the circle.

- The distance from the center to any point on the circle is the **radius** of the circle.
- A line segment with endpoints that lie on the circle is a **chord**.
- A chord that passes through the center of the circle is called a **diameter**. The diameter is the circle's longest chord. The diameter of a circle is twice as long as its radius.

PRACTICE

Use the figure below to answer Questions 1–4.



1. Name two radii.

   _____

2. Name two chords.

   _____

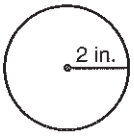
3. Name a diameter.

   _____

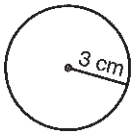
4. Name a chord that is not a diameter.

   _____

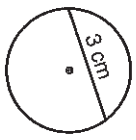
Find the diameter of each circle below. If there is not enough information, write "cannot tell."

5. 

   diameter: _____

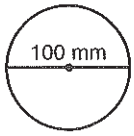
6. 

   diameter: _____

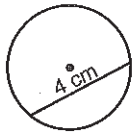
7. 

   diameter: _____

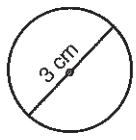
Find the radius of each circle below. If there is not enough information, write "cannot tell."

8. 

   radius: _____

9. 

   radius: _____

10. 

    radius: _____

Visualizing Shapes

In geometry, you often have to be able to imagine a figure from a different point of view. You may also need to imagine a shape to draw it.

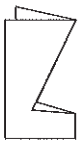
PRACTICE

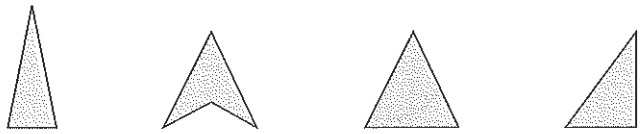
Circle the letter of the correct answer to each question.

1. This drawing shows a piece of paper that has been folded in half and then cut as shown. What is the shape of the cutout when it is unfolded?

A B C D

2. What is the shape of this cutout when it is unfolded?





F G H J

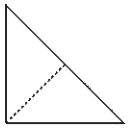
3. If you trace around the bottom of this cylinder, what shape will you draw?

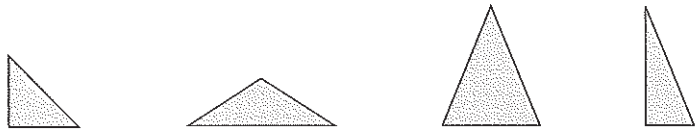
A B C D

4. If you trace around the bottom of this object, what shape will you draw?

F G H J

5. If you fold this figure along the dotted line, what is the shape of the *folded* figure?

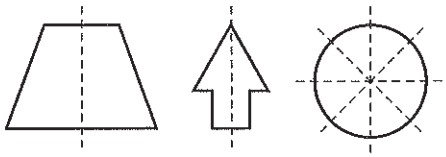




A B C D

Recognizing Symmetry

If a figure can be divided into two halves that are mirror images of one another, we say the figure has **symmetry**, or is **symmetric**. The line that separates the two halves is called the **line of symmetry**. A shape can have more than one line of symmetry.



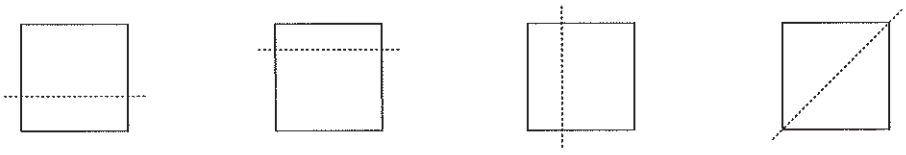
one line of symmetry

more than one line of symmetry

PRACTICE

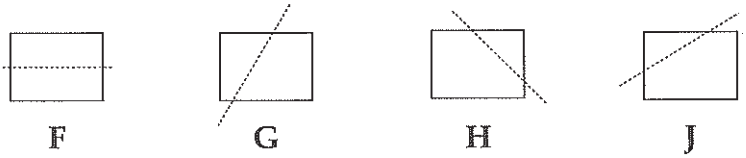
Circle the letter of the figure in each row that has been divided in half symmetrically.

1.



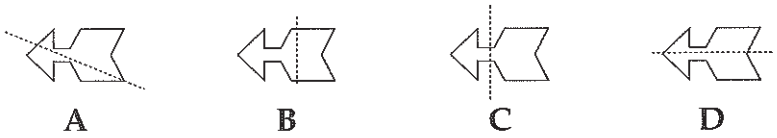
A B C D

2.



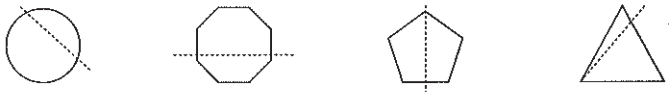
F G H J

3.



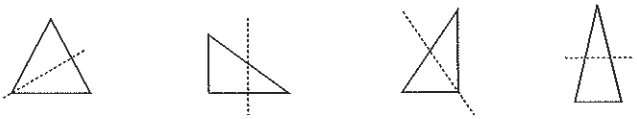
A B C D

4.



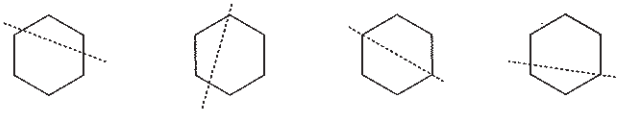
F G H J

5.



A B C D

6.



F G H J

Circle each letter below that can be divided in half symmetrically.

7. A M G Y

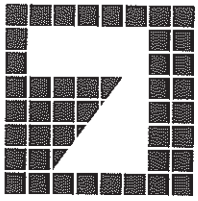
8. F J H X

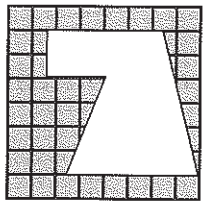
Recognizing Congruent Figures

When two figures are exactly the same size and shape, they are **congruent**. Figures do not have to be facing the same direction to be congruent.

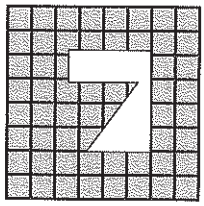
PRACTICE

For Numbers 1–3, circle the letter of the figure that is congruent to the one in the dark box.

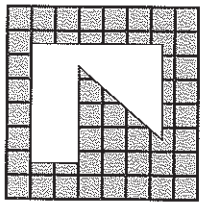
1. 



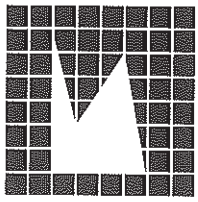
A

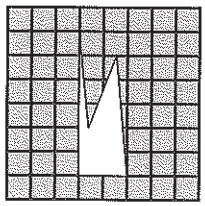


B

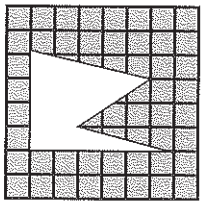


C

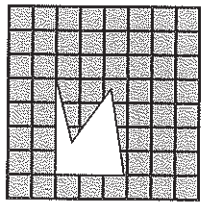
2. 



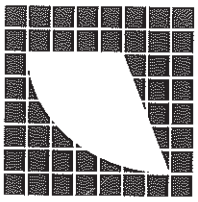
F

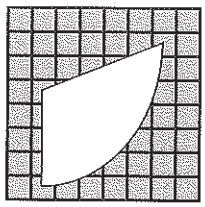


G

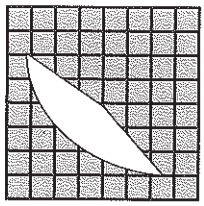


H

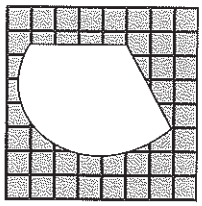
3. 



A

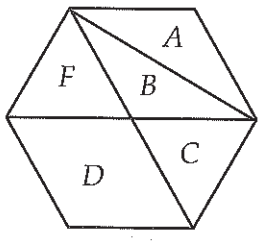


B



C

Use the diagram below for Questions 4–6.



4. Which figure is congruent to figure *A*?

_____

5. Which figure is congruent to figure *C*?

_____

6. Which 2 figures joined together form a shape that is congruent to figure *D*?

_____

Recognizing Similar Figures

When two figures have exactly the same shape, but not necessarily the same size, they are **similar**. Figures do not have to be facing the same direction to be similar.

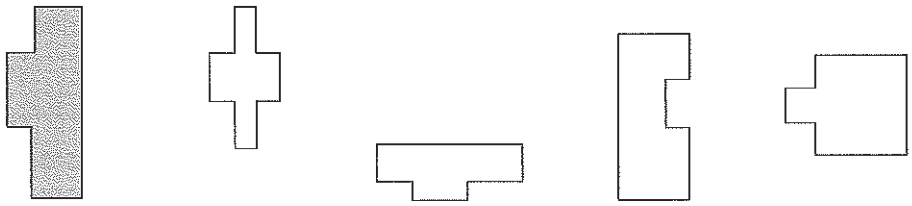
PRACTICE

For Questions 1–3, circle the letter of the shape that appears to be similar to the shaded figure.

1.

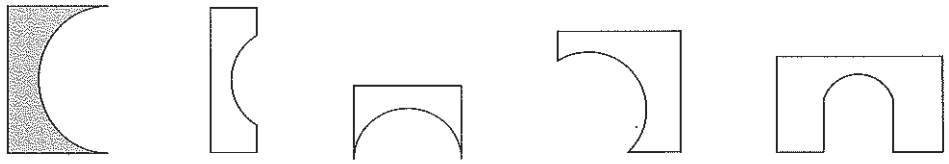
A B C D

2.



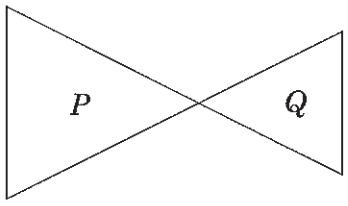
F G H J

3.



A B C D

Use the diagram below to answer Questions 4 and 5.



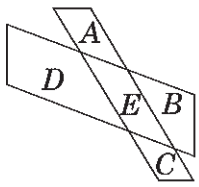
4. Do triangles P and Q look similar?

_____

5. Do triangles P and Q look congruent?

_____

Use the diagram below to answer Questions 6 and 7.



6. Which figure, if any, looks similar to figure A?

_____

7. Which figure, if any, looks similar to figure B?

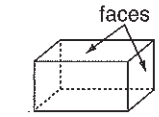
_____

Comparing Three-Dimensional Figures

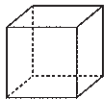
A **three-dimensional figure** has length, width, and depth. Three-dimensional figures are also called **solid figures**. A flat surface of a three-dimensional solid is called a **face**.

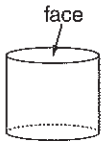
Some common three-dimensional figures are described here.

- A **rectangular solid** is the shape of a box. It has 6 faces, and each face is a rectangle. Every corner of a rectangular solid forms a right angle.
- A **cube** is a rectangular solid with 6 square faces. Every corner of a cube forms a right angle.
- A **cylinder** is shaped like a can. The top and bottom surfaces are circles.
- A **cone** has one circular face. At the opposite end, the cone comes to a point.

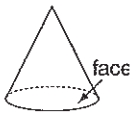


rectangular solid


cube



cylinder



cone

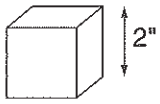
PRACTICE

Identify which shape each of the following objects looks most like. Write cube, rectangular solid, cylinder, or cone.

1. a shoebox _____
2. a roll of paper towels _____
3. each of a pair of dice _____
4. a Christmas tree _____
5. a can of frozen orange juice _____
6. a telephone book _____
7. a drinking glass _____

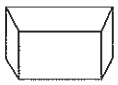
Use what you know about polygons and solids to answer the questions below.

8. What is the total area of all six faces of this cube?



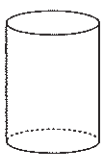
_____

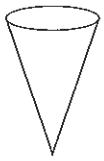
9. Is this figure a rectangular solid?



_____

10. Which would hold more water, this cylinder or this cone?





_____

Using Geometry in Solving Problems

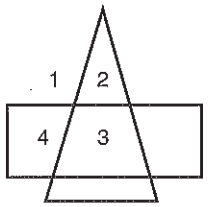
The problems on this page are different from usual math problems. They can be fun to solve, but they require that you read the question carefully and analyze the information. You can use the problem-solving steps to answer these types of questions.

- **Identify the question.**
- **Determine what information you need.**
- **Make a plan.**
- **Solve the problem.**
- **Check your work.**

PRACTICE

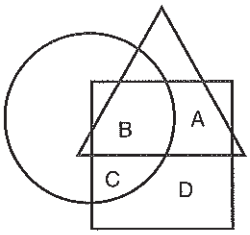
Write the letter or number that answers the question.

1. Which number is inside the rectangle but outside the triangle?



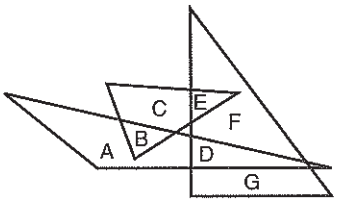
_____

2. Which letter is inside the circle and the square, but outside the triangle?



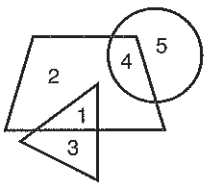
_____

3. Which letter is inside both the obtuse triangle and the acute triangle?



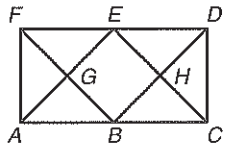
_____

4. Which number is inside only a figure that is not a polygon?



_____

Use the figure below to answer Questions 5–8.



5. Which is an obtuse angle?

A $\angle FAB$ **C** $\angle BDC$
B $\angle FEC$ **D** $\angle ECB$

6. Which is a right angle?

F $\angle ABF$ **H** $\angle EDC$
G $\angle ABG$ **J** $\angle FEG$

7. Which triangle is congruent to $\triangle AEC$?

A $\triangle EDH$ **C** $\triangle AGB$
B $\triangle FAB$ **D** $\triangle FBD$

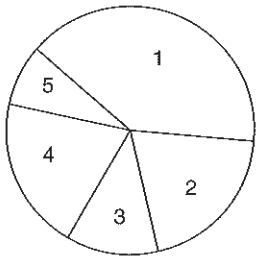
8. Which figure is *not* a parallelogram?

F figure *FECB*
G figure *DCAF*
H figure *EGBH*
J figure *ACDE*

Geometry Skills Checkup

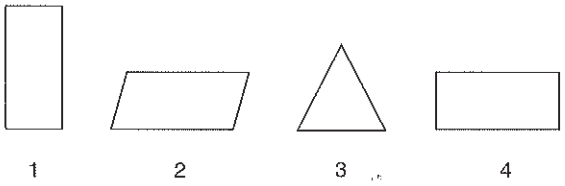
Circle the letter for the best answer to each question.

1. Which sections of this figure are congruent?



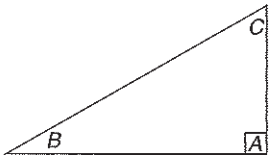
A 1 and 5
B 2 and 4
C 2 and 5
D 1 and 4

2. Which figures are similar?



F 1 and 3
G 1 and 4
H 2 and 3
J 1, 2, and 4

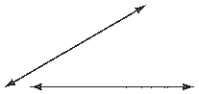
3. What is the measure of angle *A*?



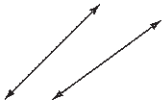
A 30 degrees
B 45 degrees
C 90 degrees
D 180 degrees

4. Which diagram shows parallel lines?

F



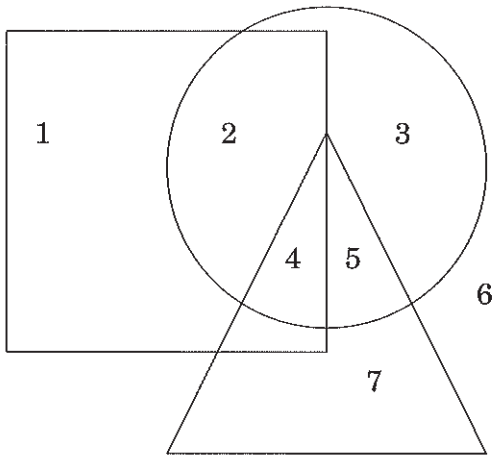
G



H

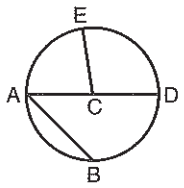
J

5. Which number is inside the square and the circle, but outside the triangle?



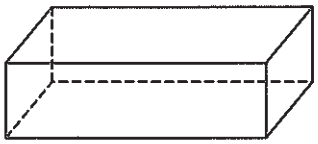
A 2
B 4
C 3
D 5

6. Which best describes $\overline{AC}$?



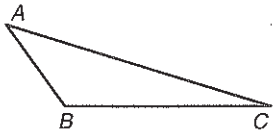
F chord
G radius
H diameter
J line

7. Which best describes this figure?



A cube
B cone
C rectangular solid
D pyramid

8. Which best describes triangle *ABC*?

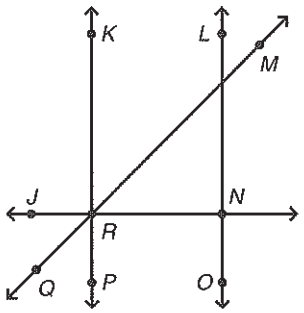


F acute and scalene
G obtuse and isosceles
H right and isosceles
J obtuse and scalene

9. Which describes the features of a regular polygon?

A equal sides but not equal angles
B equal angles but not equal sides
C neither equal sides nor equal angles
D both equal sides and equal angles

Use the drawing below to answer Questions 10–12. Circle the term that best describes the relationship between the lines named.



10. $\overleftrightarrow{QM}$ and $\overleftrightarrow{JN}$

F intersecting
G parallel
H similar
J perpendicular

11. $\overleftrightarrow{KP}$ and $\overleftrightarrow{JN}$

A intersecting
B parallel
C similar
D perpendicular

12. $\overleftrightarrow{KP}$ and $\overleftrightarrow{LO}$

F intersecting
G parallel
H similar
J perpendicular

Page 135

1. 34 ft
2. 30 mi
3. 30 in.
4. 10'
5. 8 cm
6. 22'
7. 16 ft
8. 2 cm

Page 136

1. 36 cm^2
2. 8 mm^2
3. 50 mi^2
4. 36 in^2
5. 4 ft^2
6. 4 cm^2
7. 18 m^2
8. 5 in.

Page 137

1. A, C
2. F, J
3. C
4. J
5. 30 min
6. 120 min
7. 20 min
8. 48 hrs
9. 90 min
10. 45 min

Page 138

1. 9:00
2. 3:00
3. 11:00
4. 4:20
5. 1:45
6. 1:45

Page 139

1. 5 hr 5 min
2. 3 hrs 45 min
3. 4 hrs 25 min
4. 6 hrs 5 min
5. 50 min
6. 6 hrs 20 min

Page 140

1. 5 hrs 45 min
2. 1 hr 30 min
3. 6 hrs 20 min
4. 9 hrs
5. 5 hrs 24 min
6. 2 hrs 35 min
7. 2 hrs 50 min
8. 2 hrs 35 min
9. 45 min
10. 1 hr 45 min

Measurement Skills Checkup

Pages 141–142

1. D
2. H
3. B
4. H
5. A
6. H
7. A
8. F
9. B
10. J
11. A
12. G
13. C
14. J
15. C
16. H
17. C
18. H
19. C
20. G

Geometry

Page 143

1. ray *ST*
2. line segment *GH* or *HG*
3. line *KL* or *LK*
4. line *XY* or *YX*
5. ray *MN*
6. line segment *EF* or *FE*

Page 144

1. C
2. F
3. D
4. H
5. A
6. F

Page 145

1. B, E, F, G, H
2. A, C, D
3. F, H

4. Avenue A and Oak Avenue B and Oak
5. Avenues A and B

Page 146

1. A, D, K
2. B, C, F, G, I, L
3. E, M
4. J
5. F
6. E, F, J, N
7. H
8. N

Page 147

1. A, B, F, G
2. A, F
3. C, E
4. B, F
5. True
6. True
7. True
8. True
9. True
10. True

Page 148

1. 45°, right
2. 110°, obtuse
3. 30°, obtuse
4. 50°, acute
5. right and scalene
6. acute and isosceles
7. obtuse and isosceles
8. obtuse and scalene

Page 149

1. any 2: $\overline{AC}$, $\overline{CD}$, $\overline{EC}$, $\overline{CB}$
2. $\overline{ED}$, $\overline{EB}$
3. $\overline{EB}$
4. $\overline{ED}$
5. 4 in.
6. 6 cm
7. cannot tell
8. 50 mm
9. cannot tell
10. $1\frac{1}{2}$ cm

Page 150

1. B
2. H
3. A
4. F
5. A

Page 151

1. D
2. F
3. D
4. H
5. A
6. H
7. A, M, Y
8. H, X

Page 152

1. C
2. G
3. A
4. B
5. F
6. A and B

Page 153

1. B
2. G
3. B
4. Yes.
5. No.
6. D
7. C

Page 154

1. rectangular solid
2. cylinder
3. cube
4. cone
5. cylinder
6. rectangular solid
7. cylinder
8. 24 in.^{3}
9. No.
10. cylinder

Page 155

1. 4
2. C
3. B
4. 5
5. B
6. H
7. D
8. J

Geometry Skills Checkup

Pages 156–157

1. B
2. G
3. C
4. H
5. A
6. G
7. C
8. J
9. D
10. F
11. D
12. G