



Exponential Functions

An **exponential function** is a function of the form $y = a \bullet b^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$, and the **exponent** must be a **variable**.



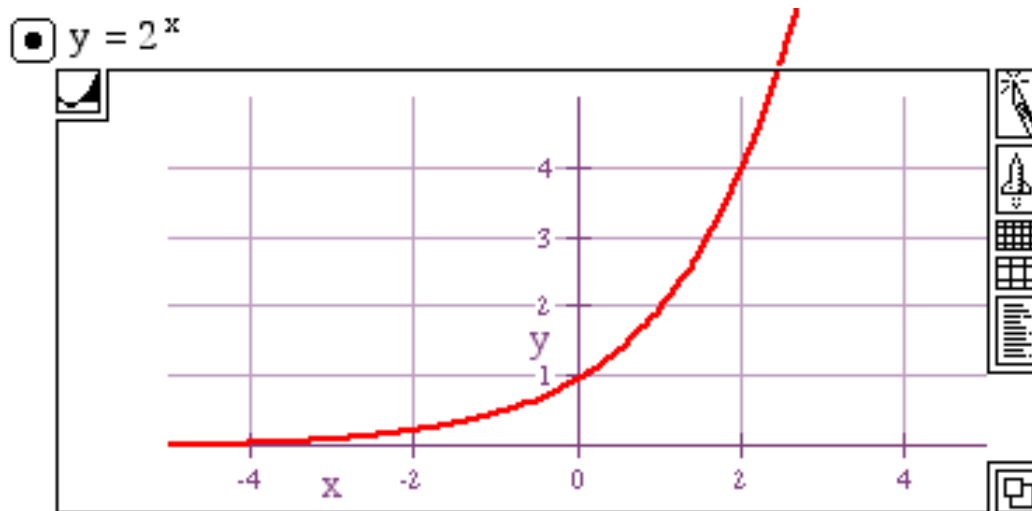
Our presentation today will consist of two sections.

Section 1: Exploration of exponential functions and their graphs.

Section 2: Discussion of the equality property and its applications.

First, let's take a look at an exponential function

$$y = 2^x$$



x	y
0	1
1	2
2	4
-1	1/2
-2	1/4


$$y = a \cdot b^x$$

So our general form is simple enough.

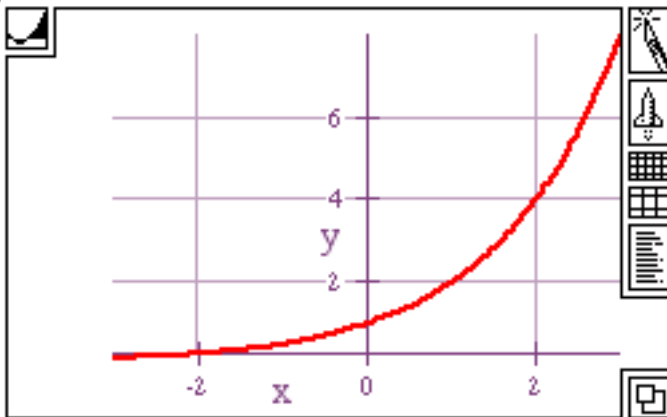
The general shape of our graph will be determined by the exponential variable.

Which leads us to ask what role does the 'a' and the base 'b' play here.

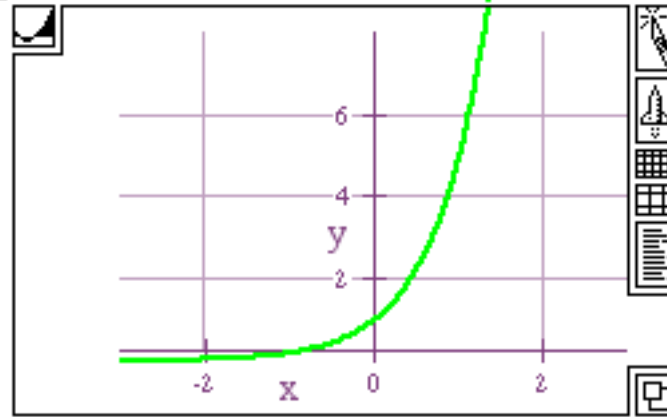
Let's take a look.

First let's change the base b
to positive values

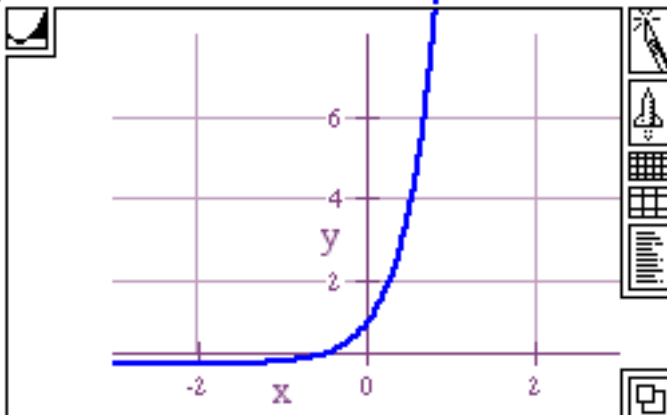
$y = 2^x$



$y = 5^x$

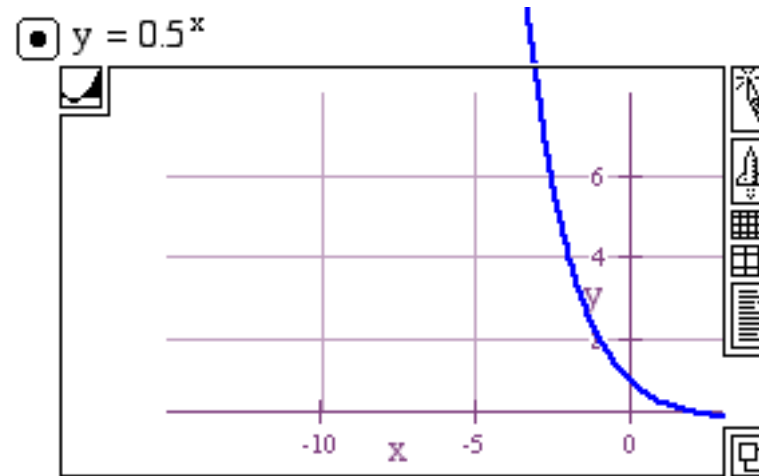
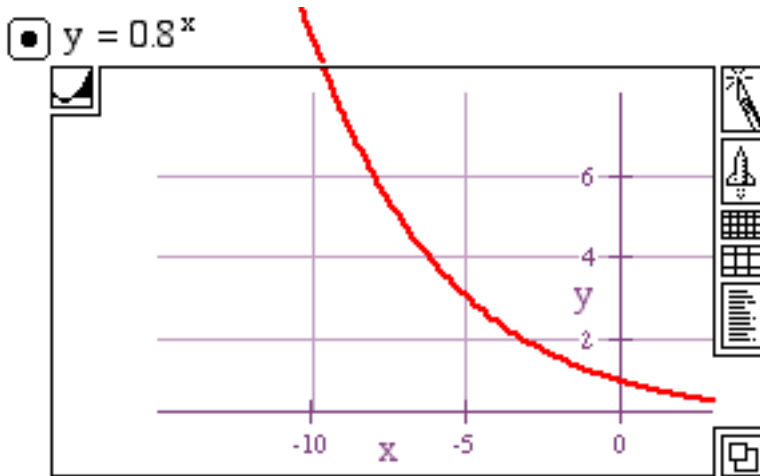


$y = 15^x$

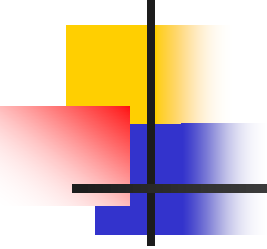


What conclusion can
we draw ?

Next, observe what happens when b assumes a value such that $0 < b < 1$.



Can you explain why this happens ?



What do you think
will happen if 'b' is
negative ?



Don't forget our definition !

Any equation of the form:

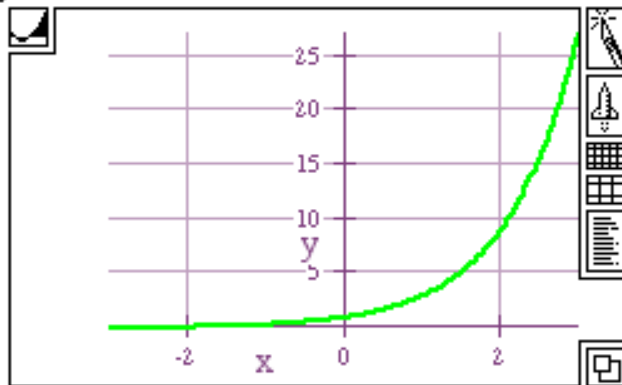
$$y = a \bullet b^x, \text{ where } a \neq 0, b > 0 \text{ and } b \neq 1$$

Can you explain why '**b**' is restricted from
assuming negative values ?

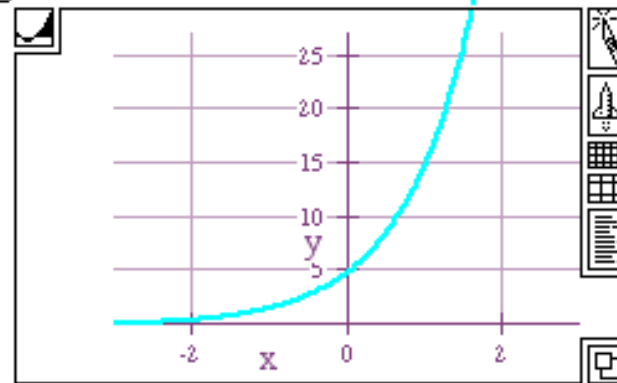

$$y = a \cdot b^x$$

To see what impact 'a' has on our graph we will fix the value of 'b' at 3.

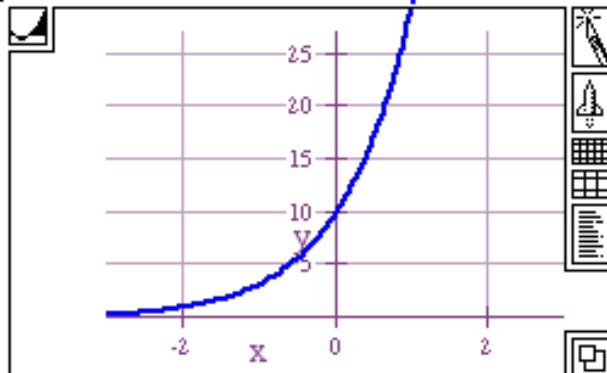
$y = 1 \cdot 3^x$



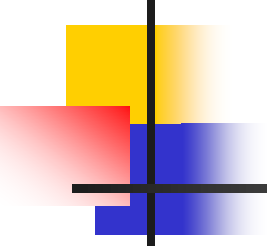
$y = 5 \cdot 3^x$



$y = 10 \cdot 3^x$



What does a larger value of 'a' accomplish ?

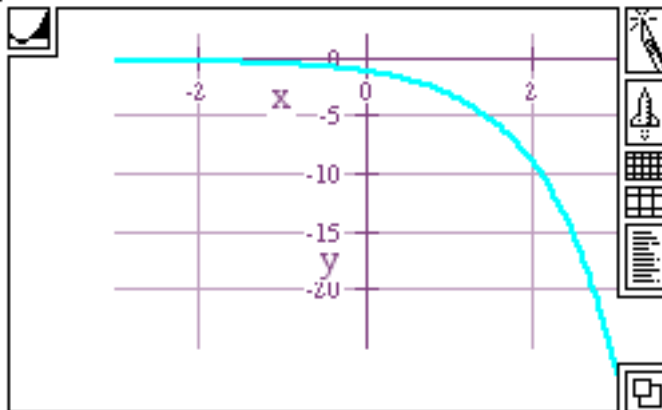


Shall we speculate as to what happens
when 'a' assumes negative values ?

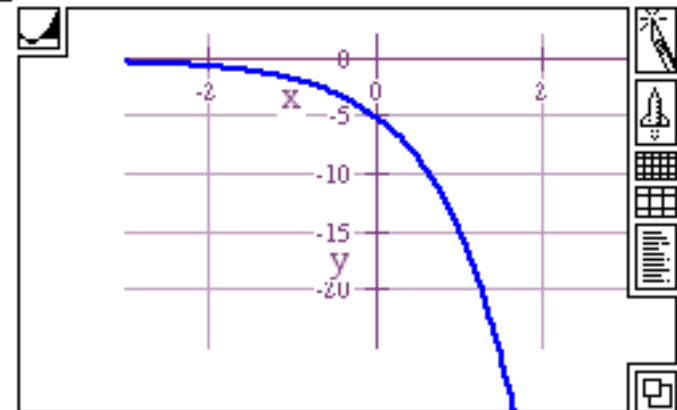
Let's see if you are correct !


$$y = a \cdot b^x \text{ where } a \neq 0, b > 0 \text{ and } b \neq 1$$

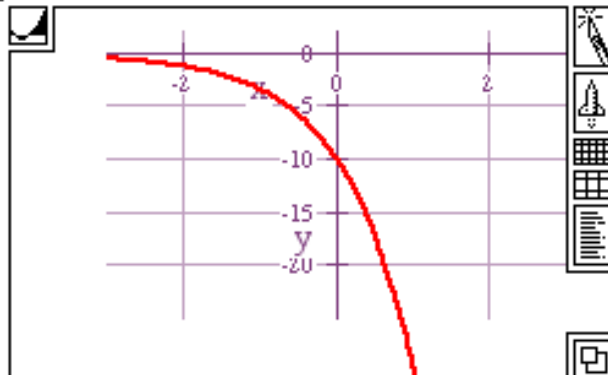
$y = -1 \cdot 3^x$

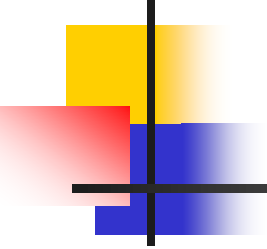


$y = -5 \cdot 3^x$



$y = -10 \cdot 3^x$



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- Our general exponential form is $y = a \bullet b^x$
 - “b” is the base of the function and changes here will result in:
 - When $b > 1$, a steep increase in the value of ‘y’ as ‘x’ increases.
 - When $0 < b < 1$, a steep decrease in the value of ‘y’ as ‘x’ increases.


$$y = a \cdot b^x$$

- We also discovered that changes in “a” would change the y-intercept on its corresponding graph.
- Now let’s turn our attention to a useful property of exponential functions.



Section 2

The Equality Property of Exponential Functions



We know that in exponential functions the exponent is a variable.

When we wish to solve for that variable we have **two** approaches we can take.

One approach is to use a logarithm. We will learn about these in a later lesson.

The second is to make use of the Equality Property for Exponential Functions.



The Equality Property for Exponential Functions

Suppose b is a positive number other than 1. Then $b^{x_1} = b^{x_2}$ if and only if $x_1 = x_2$.

Basically, this states that if the bases are the same, then we can simply set the exponents equal.

This property is quite useful when we are trying to solve equations involving exponential functions.

Let's try a few examples to see how it works.



Example 1:

$$3^{2x-5} = 3^{x+3}$$

$$2x - 5 = x + 3$$

$$x - 5 = 3$$

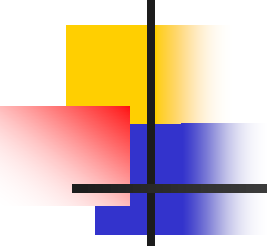
$$x = 8$$

(Since the bases are the same we simply set the exponents equal.)

Here is another example for you to try:

Example 1a:

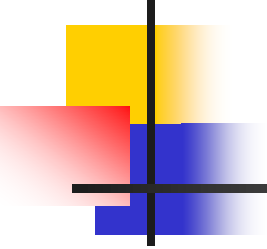
$$2^{3x-1} = 2^{\frac{1}{3}x+5}$$



The next problem is what to do when
the bases are not the same.

$$3^{2x+3} = 27^{x-1}$$

Does anyone have
an idea how
we might approach this?



Our strategy here is to rewrite the bases so that they are both the same.
Here for example, we know that

$$3^3 = 27$$



Example 2: (Let's solve it now)

$$3^{2x+3} = 27^{x-1}$$

$$3^{2x+3} = 3^{3(x-1)}$$

(our bases are now the same
so simply set the exponents equal)

$$2x + 3 = 3(x - 1)$$

$$2x + 3 = 3x - 3$$

$$-x + 3 = -3$$

$$-x = -6$$

$$x = 6$$

Let's try another one of these.



Example 3

$$16^{x+1} = \frac{1}{32}$$

$$2^{4(x+1)} = 2^{-5}$$

$$4(x+1) = -5$$

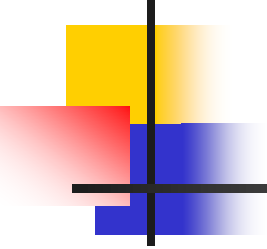
$$4x + 4 = -5$$

$$4x = -9$$

$$x = -\frac{9}{4}$$

Remember a negative exponent is simply another way of writing a fraction

The bases are now the same so set the exponents equal.



By now you can see that the equality property is actually quite useful in solving these problems. Here are a few more examples for you to try.

Example 4: $3^{2x-1} = \frac{1}{9}$

Example 5: $4^{x+3} = 8^{2x+1}$