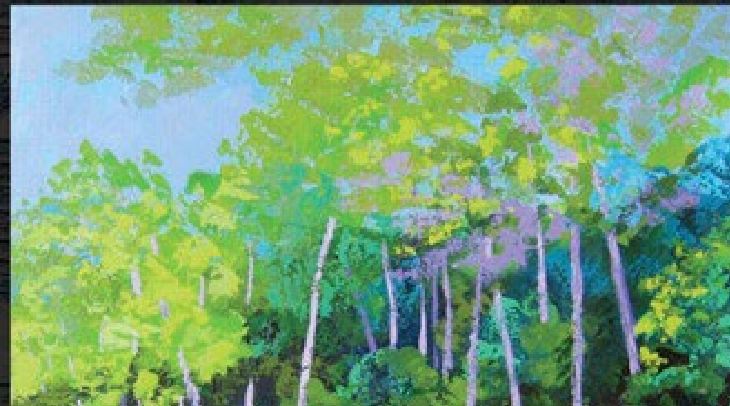
NINTH EDITION



INTERMEDIATE ALGEBRA

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Chapter 4

Graphs, Linear Equations, and Functions

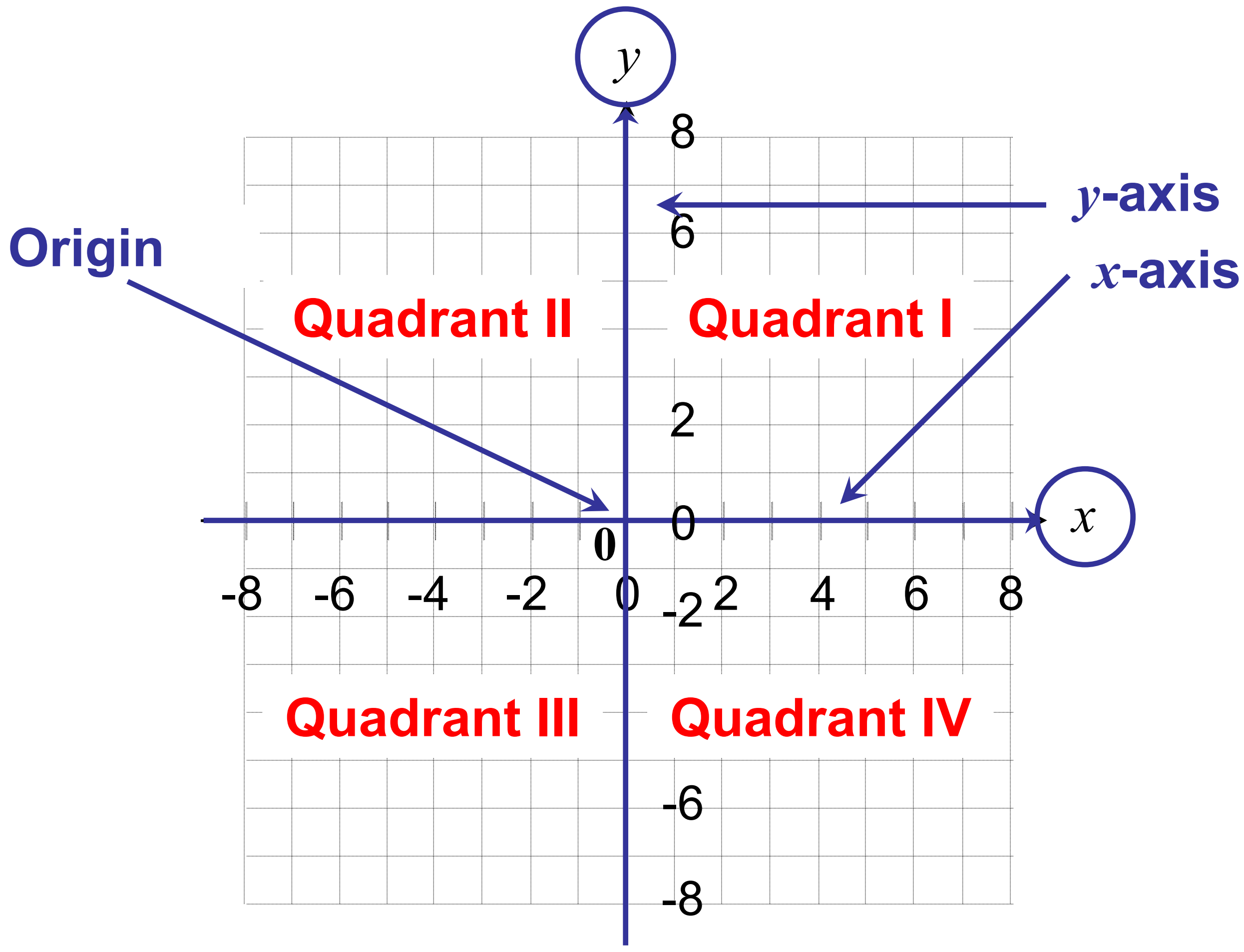
4.1

The Rectangular Coordinate System

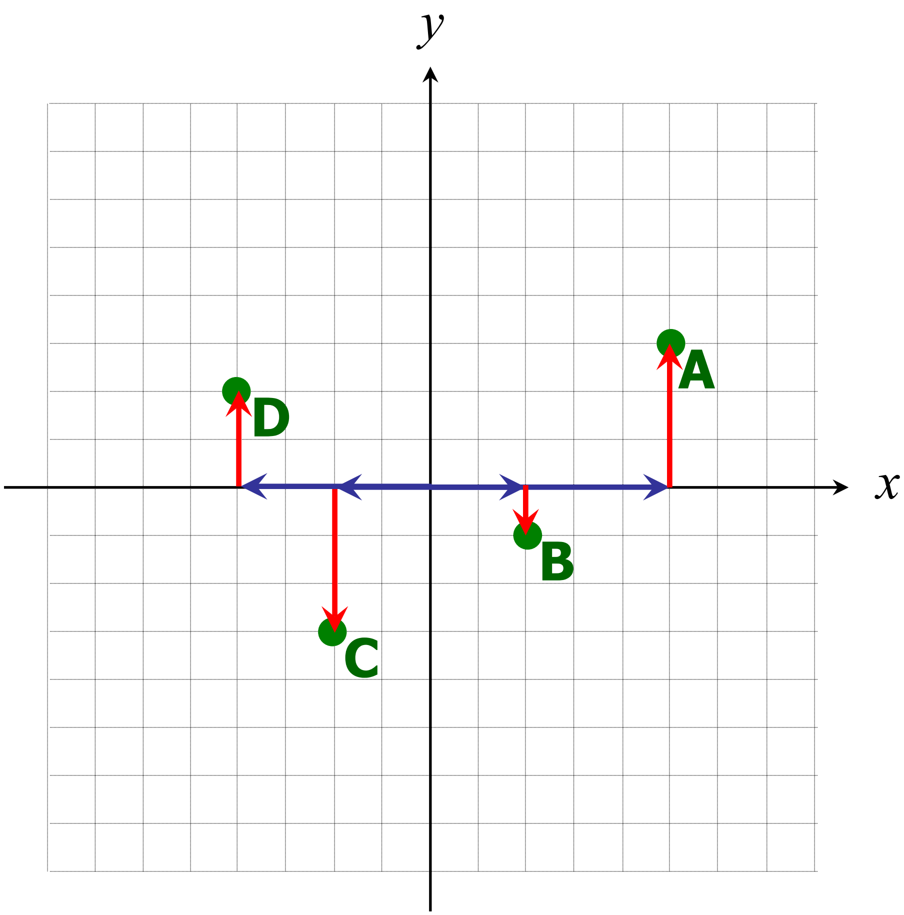
Objectives

1. Interpret a line graph.
2. Plot ordered pairs.
3. Find ordered pairs that satisfy a given equation.
4. Graph lines.
5. Find *x*- and *y*-intercepts.
6. Recognize equations of horizontal and vertical lines and lines passing through the origin.
7. Use the midpoint formula.

Rectangular (or **Cartesian,** for Descartes) **Coordinate System**



Rectangular (or **Cartesian,** for Descartes) **Coordinate System**



Ordered Pair (x, y)	Quadrant
A (5, 3)	Quadrant I
B (2, –1)	Quadrant IV
C (–2, –3)	Quadrant III
D (–4, 2)	Quadrant II

Caution

CAUTION

The parentheses used to represent an ordered pair are also used to represent an open interval (introduced in **Section 3.1**). The context of the discussion tells whether ordered pairs or open intervals are being represented.

EXAMPLE 1 Completing Ordered Pairs

Complete each ordered pair for $3x + 4y = 7$.

(a) (5, **?**)

We are given $x = 5$. We substitute into the equation to find y.

$$3x + 4y = 7$$

$$3(5) + 4y = 7 \qquad \text{Let } x = 5.$$

$$15 + 4y = 7$$

$$4y = -8$$

$$y = -2$$

The ordered pair is (**5**, **–2**).

EXAMPLE 1 Completing Ordered Pairs

Complete each ordered pair for $3x + 4y = 7$.

(b) (? , –5)

Replace y with –5 in the equation to find x.

$$3x + 4y = 7$$

$$3x + 4(-5) = 7 \qquad \text{Let } y = -5.$$

$$3x - 20 = 7$$

$$3x = 27$$

$$x = 9$$

The ordered pair is (9, –5).

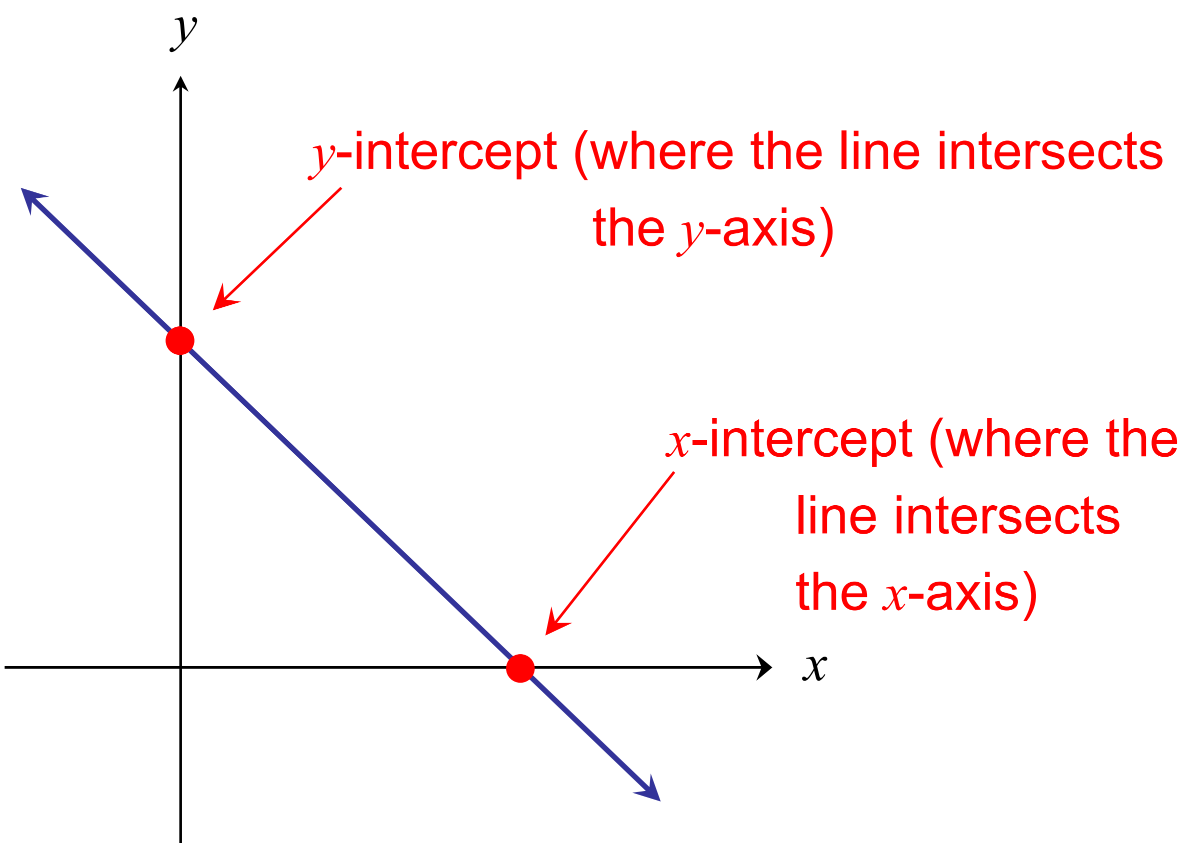
A Linear Equation in Two Variables

A **linear equation in two variables** can be written in the form

$$Ax + By = C,$$

where A, B, and C are real numbers (A and B not both 0). This form is called **standard form.**

Intercepts


y
y-intercept (where the line intersects the y-axis)
x-intercept (where the line intersects the x-axis)
x

Finding Intercepts

When graphing the equation of a line,

let $y = 0$ to find the x-intercept;

let $x = 0$ to find the y-intercept.

EXAMPLE 2 Finding Intercepts

Find the x- and y-intercepts of $2x - y = 6$, and graph the equation.

We find the x-intercept by letting $y = 0$.

$2x - y = 6$

$2x - 0 = 6$ Let $y = 0$.

$2x = 6$

$x = 3$ x-intercept is (3, 0).

We find the y-intercept by letting $x = 0$.

$2x - y = 6$

$2(0) - y = 6$ Let $x = 0$.

$-y = 6$

$y = -6$ y-intercept is (0, –6).

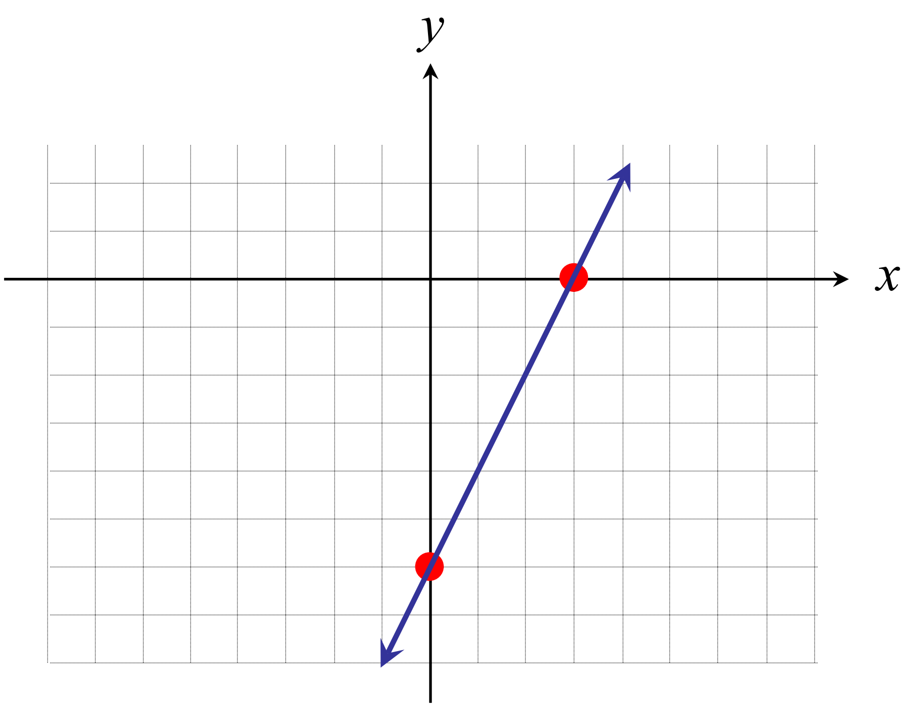
The intercepts are the two points (3,0) and (0, –6).

EXAMPLE 2 **Finding Intercepts**

Find the x- and y-intercepts of $2x - y = 6$, and graph the equation.

The intercepts are the two points (3,0) and (0, –6). We show these ordered pairs in the table next to the figure below and use these points to draw the graph.

x	y
3	0
0	–6

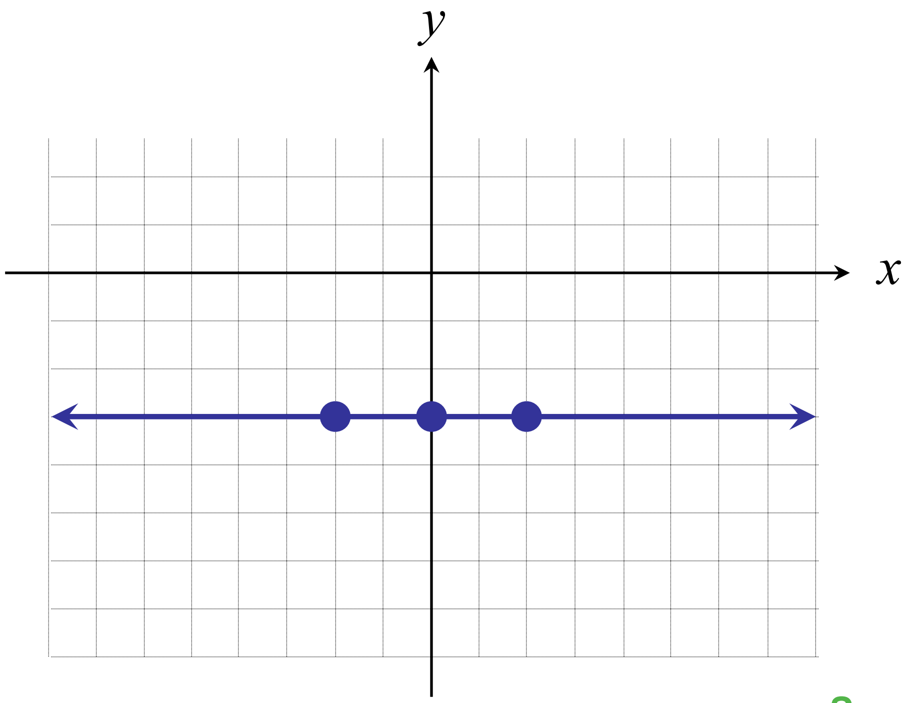


EXAMPLE 3 Graphing a Horizontal Line

Graph $y = -3$.

Since y is always –3, there is no value of x corresponding to $y = 0$, so the graph has no x-intercept. The y-intercept is (0, –3). The graph in the figure below, shown with a table of ordered pairs, is a horizontal line.

x	y
2	–3
0	–3
–2	–3

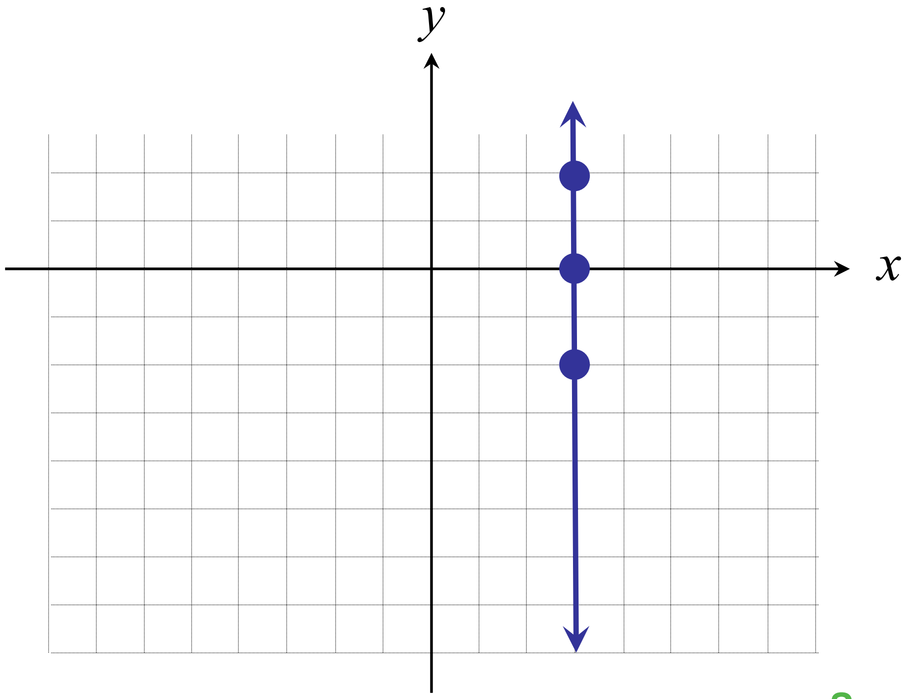


EXAMPLE 3 con't Graphing a Vertical Line

Graph $x + 2 = 5$.

The x-intercept is (3, 0). The standard form $1x + 0y = 3$ shows that every value of y leads to $x = 3$, so no value of y makes $x = 0$. The only way a straight line can have no y-intercept is if it is vertical, as in the figure below.

x	y
3	2
3	0
3	–2



Horizontal and Vertical Lines

CAUTION

To avoid confusing equations of horizontal and vertical lines remember that

1. An equation with only the variable x will always intersect the x-*axis* and thus will be *vertical*.

2. An equation with only the variable y will always intersect the y-*axis* and thus will be *horizontal*.

EXAMPLE 4 Graphing a Line That Passes through the Origin

Graph $3x + y = 0$.

We find the x-intercept by letting $y = 0$.

$3x + y = 0$

$3x + 0 = 0$ Let $y = 0$.

$3x = 0$

$x = 0$ x-intercept is (0, 0).

We find the y-intercept by letting $x = 0$.

$3x + y = 0$

$3(0) + y = 0$ Let $x = 0$.

$0 + y = 0$

$y = 0$ y-intercept is (0, 0).

Both intercepts are the same ordered pair, (0, 0). (This means the graph goes through the origin.)

EXAMPLE 4 Graphing a Line That Passes through the Origin

Graph $3x + y = 0$.

To find another point to graph the line, choose any nonzero number for x, say $x = 2$, and solve for y.

Let $x = 2$.

$$3x + y = 0$$

$$3(2) + y = 0 \qquad \text{Let } x = 2.$$

$$6 + y = 0$$

$$y = -6$$

This gives the ordered pair (2, –6).

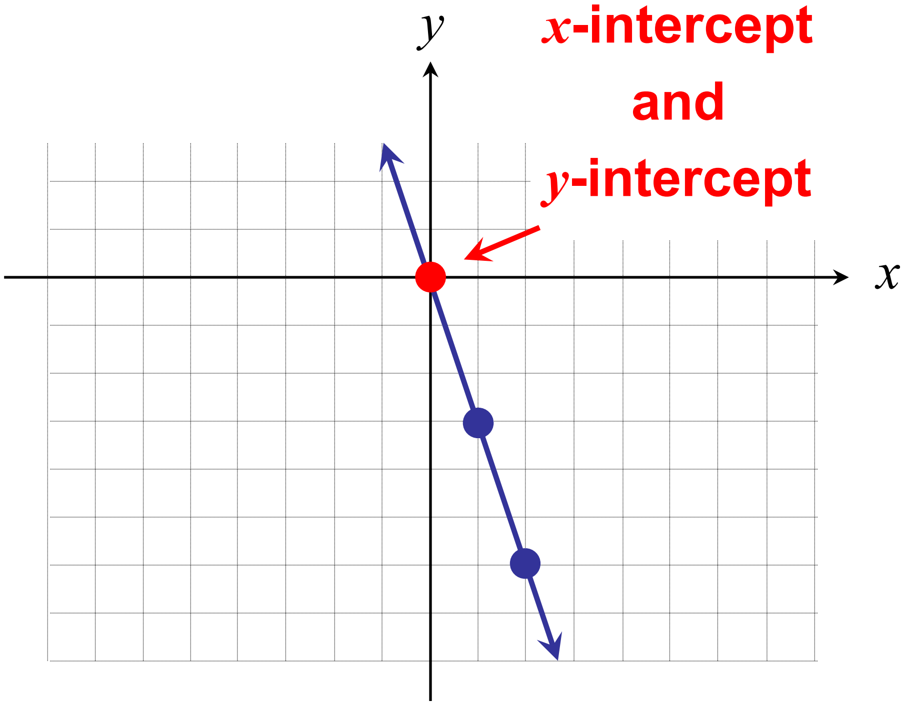
EXAMPLE 4 **Graphing a Line That Passes through the Origin**

Graph $3x + y = 0$.

These points, (0, 0) and (2, –6), lead to the graph shown below.

As a check, verify that (1, –3) also lies on the line.

x	y
0	0
2	–6
1	–3



Use the midpoint formula

If the endpoints of a line segment *PQ* are (x_1, y_1) and (x_2, y_2), its midpoint *M* is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

EXAMPLE 5 Finding the Coordinates of a Midpoint

Find the coordinates of the midpoint of line segment *PQ* with endpoints *P*(6, −1) and *Q*(4, −2).

Use the midpoint formula with $x_1 = 6$, $x_2 = 4$, $y_1 = -1$, $y_2 = -2$:

$$\left(\frac{6+4}{2}, \frac{-1+(-2)}{2}\right) = \left(\frac{10}{2}, \frac{-3}{2}\right) = \left(5, \frac{-3}{2}\right)$$

Midpoint