Exponential Functions

Objectives

- To use the properties of exponents to:
  - Simplify exponential expressions.
  - Solve exponential equations.
- To sketch graphs of exponential functions.

Exponential Functions

- A polynomial function has the basic form: $f(x) = x^3$
- An exponential function has the basic form: $f(x) = 3^x$
- An exponential function has the variable in the exponent, not in the base.
- General Form of an Exponential Function:

$$f(x) = N^x, N > 0$$

Properties of Exponents

$$A^X \cdot A^Y = A^{X+Y}$$

$$\frac{A^X}{A^Y} = A^{X-Y}$$

$$\left(A^X\right)^Y = A^{XY}$$

$$A^{-X} = \frac{1}{A^X}$$

$$\left(AB\right)^X = A^X B^X$$

$$\frac{1}{A^{-X}} = A^X$$

$$\left(\frac{A}{B}\right)^X = \frac{A^X}{B^X}$$

$$A^{X/Y} = \sqrt[Y]{A^X} = \left(\sqrt[Y]{A}\right)^X$$

Properties of Exponents

- Simplify: $2^2 \cdot 2^3 = 2^5 = 32$

$$2^2 \cdot 2^{-6} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$\left(2^3\right)^2 = 2^6 = 64$$

Properties of Exponents

- Simplify: $\left(\frac{2}{3}\right)^{-3} = \frac{2^{-3}}{3^{-3}} = \frac{3^3}{2^3} = \frac{27}{8}$

$$\frac{3^7}{3^9} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$\left(2^{1/2}\right)\left(8^{1/2}\right) = \left(2 \cdot 8\right)^{1/2} = 16^{1/2} = \sqrt{16} = 4$$

Exponential Equations

- Solve: $5^x = 125$

$$5^x = 5^3$$

$$x = 3$$

- Solve: $7^{(x-1)} = 7^{-1/2}$

$$x - 1 = -\frac{1}{2}$$

$$x = \frac{1}{2}$$

Exponential Equations

- Solve: $8^x = 2$

$$\left(2^3\right)^x = 2^1$$

$$2^{3x} = 2^1$$

$$3x = 1$$

$$x = \tfrac{1}{3}$$

- Solve: $8^x = 4$

$$\left(2^3\right)^x = 2^2$$

$$2^{3x} = 2^2$$

$$3x = 2$$

$$x = \tfrac{2}{3}$$

Exponential Equations

- Solve: $\left(\frac{1}{3}\right)^x = 27$

$$\left(3^{-1}\right)^x = 27$$

$$3^{-x} = 3^3$$

$$-x = 3$$

$$x = -3$$

- Solve: $x^{1/3} = 27$

$$\left(x^{1/3}\right)^3 = 27^3$$

$$x = 19,683$$

Not considered an exponential equation, because the variable is now in the base.

Exponential Equations

- Solve:

Not considered an exponential equation, because the variable is in the base.

$$x^{3/4} = 8$$

$$\left(x^{3/4}\right)^{4/3} = 8^{4/3}$$

$$x = \left(\sqrt[3]{8}\right)^4$$

$$x = (2)^4$$

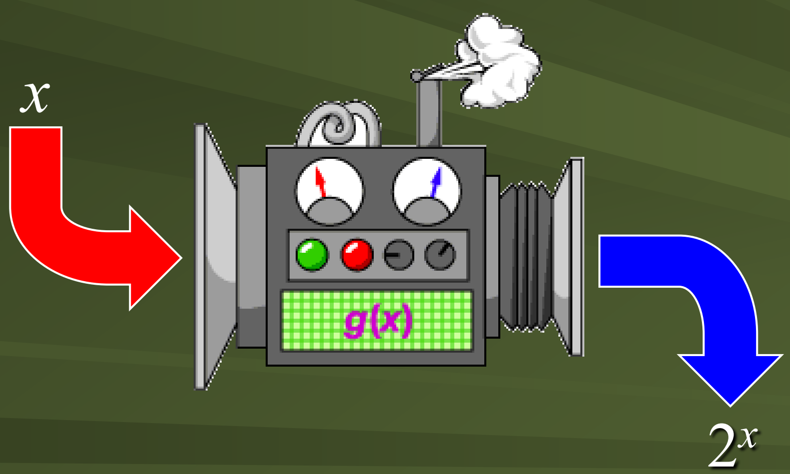
$$x = 16$$

Exponential Functions

- General Form of an Exponential Function:

$$f(x) = N^x,\ N > 0$$

$g(x) = 2^x$



$g(3) = 8$

$g(2) = 4$

$g(1) = 2$

$g(0) = 1$

$g(-1) = 2^{-1} = \frac{1}{2}$

$g(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

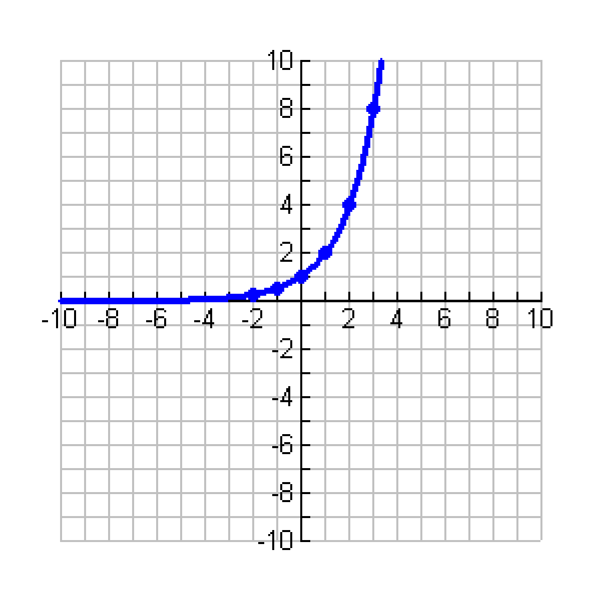
Exponential Functions

$$g(x) = 2^x$$

x	$g(x)$
2	4
1	2
0	1
–1	$\frac{1}{2}$
–2	$\frac{1}{4}$

Exponential Functions

$$g(x) = 2^x$$



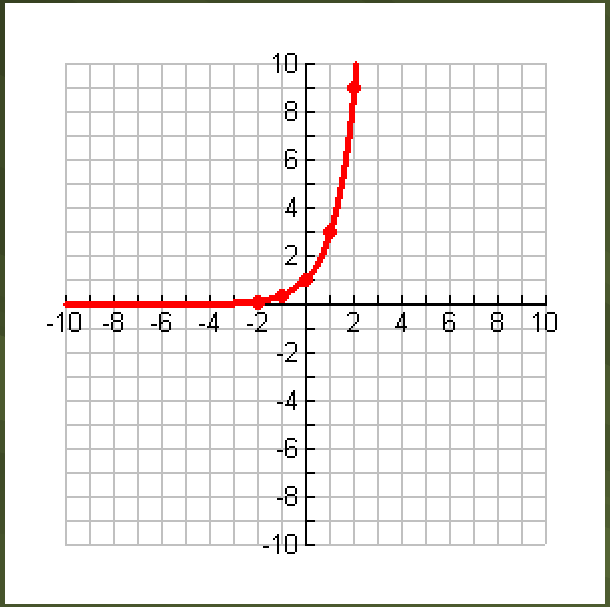
Exponential Functions

$$h(x) = 3^x$$

x	$h(x)$
2	9
1	3
0	1
–1	$\frac{1}{3}$
–2	$\frac{1}{9}$

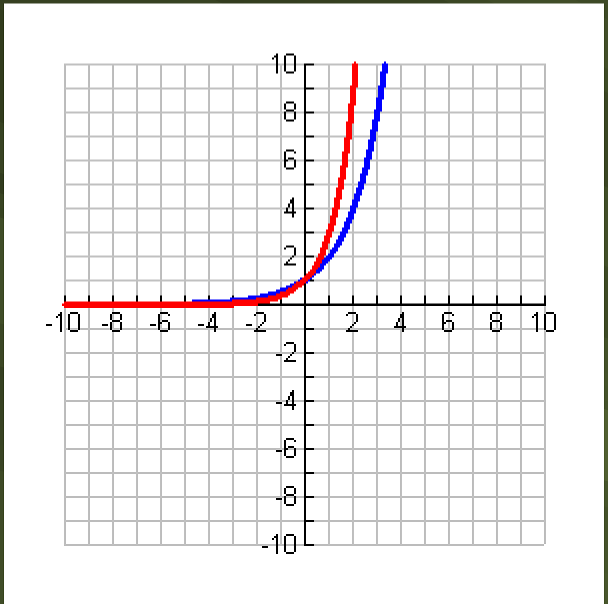
Exponential Functions

$$h(x) = 3^x$$



Exponential Functions

$g(x) = 2^x$ (blue)

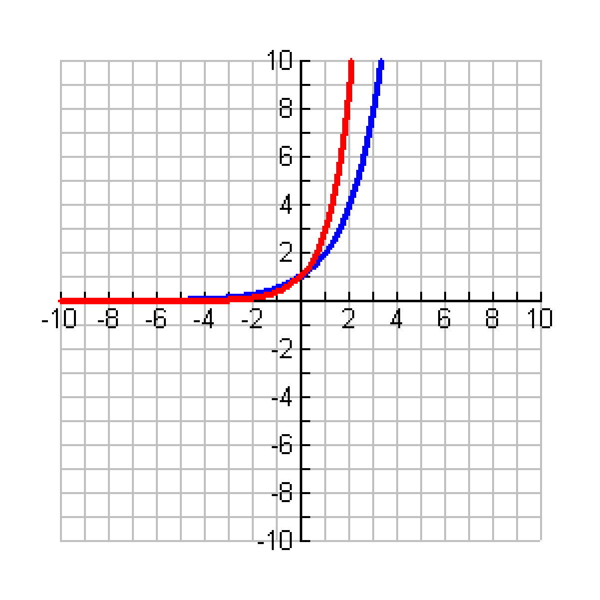


$h(x) = 3^x$ (red)

- Exponential functions with positive bases <u>greater</u> than 1 have graphs that are <u>increasing</u>.
- The function never crosses the x-axis because there is nothing we can plug in for x that will yield a zero answer.
- The x-axis is a left horizontal asymptote.

Exponential Functions

$g(x) = 2^x$ (blue)



$h(x) = 3^x$ (red)

- A smaller base means the graph rises more gradually.
- A larger base means the graph rises more quickly.
- Exponential functions will not have negative bases.

The Number e

A base often associated with exponential functions is:

$$e \approx 2.71828169$$

The Number e

- Compute: $\lim_{x\to 0}(1+x)^{1/x} \approx 2.71828169$

$$\lim_{x\to 0^-}(1+x)^{1/x}$$

x	$(1+x)^{1/x}$
–.1	2.868
–.01	2.732
–.001	2.7196

$$\lim_{x\to 0^+}(1+x)^{1/x}$$

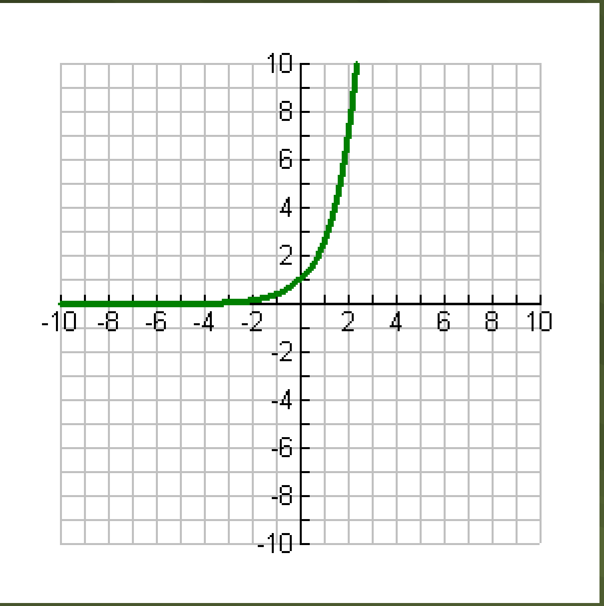
x	$(1+x)^{1/x}$
.1	2.5937
.01	2.7048
.001	2.7169

The Number *e*

- Euler's number
- Leonhard Euler (pronounced "oiler")
- Swiss mathematician and physicist

The Exponential Function

$$f(x) = e^x$$



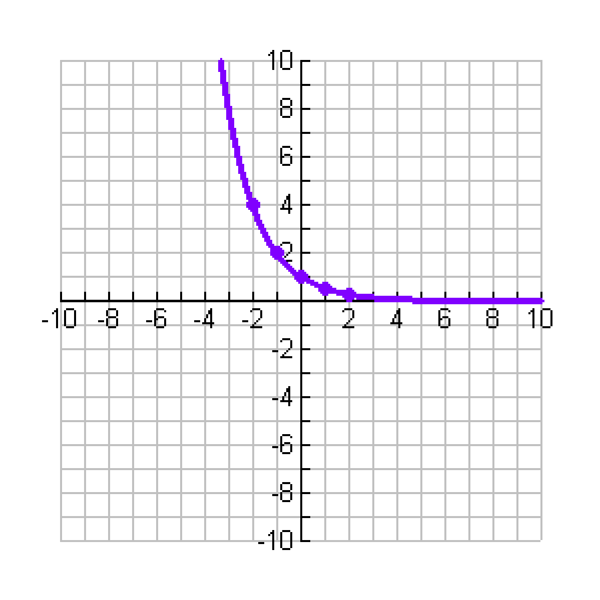
Exponential Functions

$$j(x) = \left(\tfrac{1}{2}\right)^x$$

x	$j(x)$
2	$\frac{1}{4}$
1	$\frac{1}{2}$
0	1
–1	2
–2	4

Exponential Functions

$$j(x) = \left(\tfrac{1}{2}\right)^x$$



- Exponential functions with positive bases <u>less</u> than 1 have graphs that are <u>decreasing</u>.

Why study exponential functions?

- Exponential functions are used in our real world to measure growth, interest, and decay.

- Growth obeys exponential functions.
- Ex: rumors, human population, bacteria, computer technology, nuclear chain reactions, compound interest

- Decay obeys exponential functions.
- Ex: Carbon-14 dating, half-life, Newton's Law of Cooling