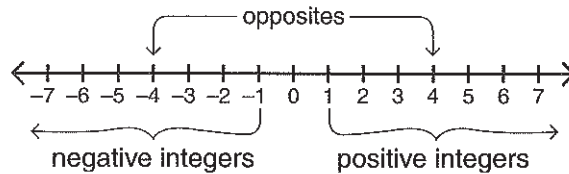# Integers

Understanding Integers

Until now we have worked with **positive numbers**, numbers that are greater than zero. Each positive number has an opposite **negative number**.

You are probably familiar with negative numbers being used to describe temperatures below zero. Negative numbers are identified by placing a minus sign in front of them. For example, 2 degrees below zero is written as $-2°$. A positive number may have a plus sign in front of it, but if there is no sign, the number is understood to be positive.

Positive whole numbers and their opposite negative whole numbers, together with zero, are called **integers**.

- As you move to the left on the number line, the value of each number decreases.
   -7 is to the left of -5, so -7 is less than -5. $-7 < -5$
   You can think about it this way: $-7°$ is colder than $-5°$, or, if you spend 7 dollars, you will have less money left than if you spend only 5 dollars.

- As you move to the right on the number line, the value increases.
   1 is to the right of -8, so 1 is greater than -8. $1 > -8$

PRACTICE

Compare the numbers in each pair. Write $<$, $>$, or $=$.

1. $-5 \bigcirc 3$

2. $-2 \bigcirc -1$

3. $0 \bigcirc -6$

4. $\dfrac{1}{2} \bigcirc -\dfrac{3}{4}$

5. $-7 \bigcirc -3$

6. $-1 \bigcirc 6$

7. $-0.02 \bigcirc -0.025$

8. $-0.5 \bigcirc -\dfrac{1}{2}$

Write the numbers in each set in order from least to greatest.
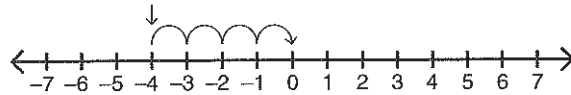
9. $5, -12, 3, 0$

10. $12, 4, -52, -13$

11. $-\dfrac{1}{2}, 3, -1.5, \dfrac{3}{4}$

Working with Absolute Value

The **absolute value** of a number is its distance from 0 on the number line.

Example
The number 4 is four spaces from 0. It has an absolute value of 4.

- The symbol for absolute value is $|\ |$.

    The absolute value of 4 is written $|4|$.

    The absolute value of -20 is written $|-20|$.

- A number's absolute value is always a positive number.

Examples

$$|12| = 12 \qquad |-3| = 3 \qquad \left|-\tfrac{2}{3}\right| = \tfrac{2}{3} \qquad |0.8| = 0.8$$

PRACTICE

Write the absolute value.

1. $|3.1| = $ _____

2. $|-3| = $ _____

3. $|5| = $ _____

4. $|-5| = $ _____

5. $|-6| = $ _____

6. $|-0.25| = $ _____

7. $|32| = $ _____

8. $|0| = $ _____

Compare the values in each pair. Write $<$, $>$, or $=$.

9. $|32| \bigcirc |-32|$

10. $|-12| \bigcirc |-15|$

11. $\left|\tfrac{1}{4}\right| \bigcirc |-0.5|$

12. $|-5| \bigcirc |5|$

13. $|22| \bigcirc |-6|$

14. $|3| \bigcirc |-8|$

15. $|16.1| \bigcirc |-20|$

16. $|1| \bigcirc |1.0|$

17. $|-4| \bigcirc 4$

18. $0 \bigcirc |-2|$

19. $|3| \bigcirc -9$

20. $|12| \bigcirc 15$

21. $-6 \bigcirc |-10|$

22. $|0.4| \bigcirc 0.35$

23. $|-6| \bigcirc 6$

24. $3 \bigcirc |-1|$

Adding with Integers

You can use absolute values to find the sums of positive and negative numbers.
- When you add two numbers with the same sign (both positive or both negative), find the sum of their absolute values. Then give the sum their common sign.

Examples   Add 3 + 5.   Add −3 + −5.
$|3| + |5| = 3 + 5 = 8$ $|-3| + |-5| = 3 + 5 = 8$
Both addends are positive.   Both addends are negative.
The sum is positive.   The sum is negative.
$3 + 5 = 8$ $-3 + -5 = -8$
$(+) + (+) = (+)$ $(-) + (-) = (-)$

- When you add two numbers with unlike signs, **find the difference in their absolute values**. Then give that difference the sign of the addend with the greater absolute value.

Examples   Add 2 + −2.   Add 4 + −3.
$|2| - |-2| = 2 - 2 = 0$ $|4| - |-3| = 4 - 3 = 1$
Zero is not positive or negative.   As $|4| > |-3|$, the sum will be positive.
*2 + −2 = 0 $4 + -3 = 1$

*Note: A number plus its opposite will always equal zero.

You can also use a number line to help you find sums when you are adding two numbers that have unlike signs. Use the first addend as your starting point.
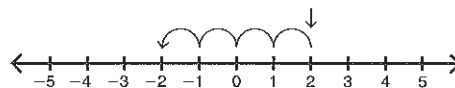
- If you are adding a positive number, move to the **right**.
- If you are adding a negative number, move to the **left**.

Examples   Add −1 + 5.   Add 2 + −4.
Start at −1 and move to the right 5.   Start at 2 and move to the left 4.
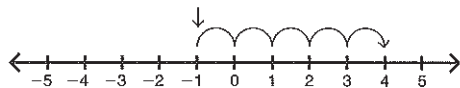
−1 + 5 = 4   2 + −4 = −2

- When there are three or more addends, add two at a time. The order does not matter.

PRACTICE

Fill in each blank. Use absolute values or a number line to find sums.

1. What is 3 to the left of 0? _____

2. What is 5 to the right of −1? _____

3. What is 3 to the left of 2? _____

4. What is 6 to the right of −5? _____

5. −12 + 7 = _____

6. 1 + −3 = _____

7. −6 + 6 = _____

8. −1 + −1 = _____

9. 7 + −2 = _____

10. −10 + −5 = _____

11. The temperature was −2°F. Then it dropped 2 degrees. What was the temperature then?

_____

12. To add −4 to 5 on a number line, start at +5 and move 4 spaces to the _____.

Subtracting with Integers

When you subtract a positive number, you take that amount away from the starting amount. The greater the number you subtract, the less you have left. And if you subtract a number greater than the starting amount, you will wind up with a negative number. However, when you subtract a negative number, it is the same as adding the number's opposite. It is possible to end up with a number that is greater than the one you started with. Study the patterns in the following examples.

Examples

$$3 - 5 = -2 \longrightarrow 3 + -5 = -2$$
$$3 - 4 = -1 \longrightarrow 3 + -4 = -1$$
$$3 - 3 = 0 \longrightarrow 3 + -3 = 0$$
$$3 - 2 = 1 \longrightarrow 3 + -2 = 1$$
$$3 - 1 = 2 \longrightarrow 3 + -1 = 2$$
$$3 - 0 = 3 \longrightarrow 3 + 0 = 3$$
$$3 - (-1) = 4 \longrightarrow 3 + 1 = 4$$

$$-4 - 2 = -6 \longrightarrow -4 + -2 = -6$$
$$-4 - 1 = -5 \longrightarrow -4 + -1 = -5$$
$$-4 - 0 = -4 \longrightarrow -4 + 0 = -4$$
$$-4 - (-1) = -3 \longrightarrow -4 + 1 = -3$$
$$-4 - (-2) = -2 \longrightarrow -4 + 2 = -2$$
$$-4 - (-3) = -1 \longrightarrow -4 + 3 = -1$$
$$-4 - (-4) = 0 \longrightarrow -4 + 4 = 0$$
$$-4 - (-5) = 1 \longrightarrow -4 + 5 = 1$$

To subtract with integers:
- Rewrite the subtraction problem as an addition problem in which you **add the opposite** of the number being subtracted. Remember, a number's opposite has the same absolute value, but it has the opposite sign. For example, the opposite of 6 is -6, and the opposite of -8 is 8.
- Then follow the rules for adding integers.

PRACTICE

Write the opposite of each number.

1. -3 _____

2. 17 _____

3. -19 _____

4. -12 _____

5. 11 _____

6. 3 _____

Rewrite each subtraction problem as an addition problem in which you add the opposite of the number being subtracted.

7. $-7 - 4 \longrightarrow$ _____

8. $14 - 6 \longrightarrow$ _____

9. $-8 - (-4) \longrightarrow$ _____

10. $3 - (-9) \longrightarrow$ _____

11. $5 - 12 \longrightarrow$ _____

12. $13 - (-2) \longrightarrow$ _____

Find each difference. Be sure to rewrite each subtraction problem to an addition problem in which you add the opposite.

13. $2 - (-3) =$ _____

14. $-3 - 2 =$ _____

15. $-4 - (-5) =$ _____

16. $2 - 5 =$ _____

17. $3 - (-3) =$ _____

18. $4 - (-3) =$ _____

19. $-7 - (-2) =$ _____

20. $-9 - 1 =$ _____

21. $0 - (-9) =$ _____

Multiplying and Dividing with Integers

Here are the rules for multiplying with positive and negative numbers.

- When you multiply two factors that have the same sign, the product is positive.

positive times positive equals positive	$(+) \times (+) = (+)$	$3 \times 5 = 15$
negative times negative equals positive	$(-) \times (-) = (+)$	$-3 \times -5 = 15$

- When you multiply two factors that have unlike signs, the product is negative.

negative times positive equals negative	$(-) \times (+) = (-)$	$-3 \times 5 = -15$
positive times negative equals negative	$(+) \times (-) = (-)$	$3 \times -5 = -15$

  If there are more than two factors, multiply two at a time. Their order does not matter.

The rules for dividing positive and negative numbers are the same as those for multiplying.

- When you divide with two numbers that have the same sign, the quotient is positive.

positive divided by positive equals positive	$(+) \div (+) = (+)$	$15 \div 5 = 3$
negative divided by negative equals positive	$(-) \div (-) = (+)$	$-15 \div -5 = 3$

- When you divide with two numbers that have unlike signs, the quotient is negative.

negative divided by positive equals negative	$(-) \div (+) = (-)$	$-15 \div 5 = -3$
positive divided by negative equals negative	$(+) \div (-) = (-)$	$15 \div -5 = -3$

PRACTICE

Find each product.

1. $2 \times -3 =$ _____

2. $-5 \times -4 =$ _____

3. $-3 \times 4 =$ _____

4. $4 \times -8 =$ _____

5. $6 \times -5 =$ _____

6. $-8 \times -8 =$ _____

7. $-7 \times -5 =$ _____

8. $-2 \times 9 =$ _____

9. $6 \times -4 =$ _____

Find each quotient.

10. $12 \div -3 =$ _____

11. $16 \div -2 =$ _____

12. $-20 \div -2 =$ _____

13. $-14 \div 7 =$ _____

14. $50 \div -25 =$ _____

15. $-32 \div -8 =$ _____

16. $-35 \div -7 =$ _____

17. $42 \div -6 =$ _____

18. $-64 \div 4 =$ _____

Simplify each fraction.

19. $\dfrac{-2}{-6} =$

20. $\dfrac{4}{-8} =$

21. $\dfrac{-6}{9} =$

Using Integers to Solve Word Problems

The problems below involve addition, subtraction, multiplication, or division of signed numbers. Set up and solve each problem. *Hint:* Some of these are two-step problems.

1. Lynn had the following transactions on her credit card: −$62.91, −$9.50, −$3.10, +$3.10. What was the change to her credit card balance?

_____

2. At 8:00 A.M. the temperature was −4°C. By noon, the temperature had risen 12°, and by 4:00 P.M. it had dropped 7° from the noon temperature. What was the temperature at 4:00 P.M.?

_____

3. A diver rested at −340 feet. Then she started descending, and 20 minutes later she was at −420 feet. How many feet per minute did she descend?

_____

4. The elevation of La Paz, Bolivia, is 12,000 feet above sea level. Death Valley, California, has an elevation of −242 feet. How much higher is La Paz than Death Valley?

_____

5. Find the current balance for this balance sheet.

Previous balance	+$246
Deposit	+$12
Expense	−$90
Current balance	

6. Last month, the books for Joe's restaurant had a balance of $27,000. But this month Joe has lost business because of street repairs being made in front of his restaurant. In the past three weeks, Joe recorded −$800, −$1,200, and −$920. What is his balance now?

_____

7. Nick has turned on his new freezer. The temperature inside is now 10°F. If the temperature drops 2 degrees each minute, how long will it take the temperature to reach −10°F?

_____

8. The XYZ Company lost $12,000 in the first quarter of the year. They made a profit of $4,650 in each of the next two quarters. They made $5,025 in the fourth quarter. What were their profits or losses for the year?

_____

9. A number multiplied −32 is 256. What is the number?

_____

10. An elevator at the 25th floor went down 16 floors and then up 11 floors. How far from the 25th floor was it?

_____

Integers Skills Checkup

Circle the letter for the correct answer to each problem.

1. $\dfrac{-12}{-24}$

   A   2   C $-\dfrac{1}{2}$

   B $\dfrac{1}{2}$ D -2

   E   None of these

2. $6 + {}^-7 =$

   F   13   H   1
   G -13 J -1
   K   None of these

3. $-12 \div {}^-2 =$

   A   6   C   24
   B -6 D -24
   E   None of these

4. $|\,5 - 9\,| =$

   F   4   H -4
   G   14   J -14
   K   None of these

5. $9 + {}^-3 + {}^-1 =$

   A   5   C   7
   B -5 D -7
   E   None of these

6. Which set of numbers is in order from least to greatest?

   A -12 0   5 -30
   B -30 -12 0   5
   C   0   5 -12 -30
   D -12 -30 0   5

7. What number is 2 less than -13?

   F -11 H -15
   G   11   J   15

8. Which of the following is greater than $\dfrac{1}{-3}$?

   A $\dfrac{1}{-2}$ C   0

   B -1 D $\dfrac{2}{-3}$

9. Lou had a $50 credit in his credit card account. Then he spent $167. Which of these number sentences could you use to find Lou's new balance?

   F $-\$50 + (-\$167) = n$
   G $\$167 + \$50 = n$
   H $\$50 + (-\$167) = n$
   J $-\$50 + \$167 = n$

10. Which of the following expressions has the same value as $-2 - (-6)$?

   A $-2 + 6$ C $-2 - 6$
   B $2 + {}^-6$ D $2 - 6$

Integers Skills Checkup (continued)

11. $10 - 3 =$

    F 13   H 7
    G -13   J -7
    K None of these

12. $-2 - (-9) =$

    A 7   C 11
    B -7   D -11
    E None of these

13. $|3| - |-5| =$

    F 2   H -2
    G 8   J -8
    K None of these

14. $-8 + -3 + 4 =$

    A 0   C 8
    B -8   D -7
    E None of these

15. $\dfrac{80}{-10} =$

    F 8   H $-\dfrac{1}{8}$
    G $\dfrac{1}{8}$ J -8
    K None of these

16. What is the distance between -3 and 7 on a number line?

    F 4   H -4
    G 10   J 11

17. Which of these decimals is less than -1?

    A -0.5   C -0.3
    B 0.75   D -2.12

18. Which of the following has the same value as $28 - 52$?

    F $-28 + 52$
    G $28 - (-52)$
    H $-28 + -52$
    J $28 + -52$

19. Which of the following does *not* equal -12?

    A 12×-1 C -12×1
    B -12×-1 D $\dfrac{-12}{1}$

20. Which product is the greatest?

    F -8×-12
    G 3×-40
    H -200×0
    J 20×-4

Fractions Skills Checkup
Pages 71–72
1. B
2. J
3. C
4. K
5. C
6. K
7. C
8. H
9. B
10. H
11. D
12. F
13. D
14. H
15. C
16. A
17. H
18. C
19. G
20. C

Integers
Page 73
1. <
2. <
3. >
4. >
5. <
6. <
7. >
8. =
9. $^-12, 0, 3, 5$
10. $^-52, ^-13, 4, 12$
11. $^-1.5, -\frac{1}{2}, \frac{3}{4}, 3$

Page 74
1. 3.1
2. 3
3. 5
4. 5
5. 6
6. 0.25
7. 32
8. 0
9. =
10. <
11. <
12. =
13. >
14. <
15. <
16. =
17. =
18. <
19. >
20. <
21. <
22. >
23. =
24. >

Page 75
1. $^-3$
2. 4
3. $^-1$
4. 1
5. $^-5$
6. $^-2$
7. 0
8. $^-2$
9. 5
10. $^-15$
11. $^-4°$ F
12. left

Page 76
1. 3
2. $^-17$
3. 19
4. 12
5. $^-11$
6. $^-3$
7. $^-7 + ^-4$
8. $14 + ^-6$
9. $^-8 + 4$
10. $3 + 9$
11. $5 + ^-12$
12. $13 + 2$
13. 5
14. $^-5$
15. 1
16. $^-3$
17. 6
18. 7
19. $^-5$
20. $^-10$
21. 9

Page 77
1. $^-6$
2. 20
3. $^-12$
4. $^-32$
5. $^-30$
6. 64
7. 35
8. $^-18$
9. $^-24$
10. $^-4$
11. $^-8$
12. 10
13. $^-2$
14. $^-2$
15. 4
16. 5
17. $^-7$
18. $^-16$
19. $\frac{1}{3}$
20. $-\frac{1}{2}$
21. $-\frac{2}{3}$

Page 78
1. $^-\$72.41$
2. 1°C
3. 4 ft
4. 12,242 ft
5. $168
6. $24,080

7. 10 min
8. $2,325 profit
9. $^-8$
10. 5 floors

Integers Skills Checkup
Page 79 – 80
1. B
2. J
3. A
4. F
5. A
6. B
7. H
8. C
9. H
10. A
11. H
12. A
13. H
14. D
15. J
16. G
17. D
18. J
19. B
20. F

Ratio, Proportion, and Percent
Page 81
1. $\dfrac{\text{inches}}{\text{miles}} = \dfrac{1}{25}$
2. $\dfrac{\$}{\text{months}} = \dfrac{2{,}958}{1}$
3. $\dfrac{\text{evergreens}}{\text{leafy}} = \dfrac{1}{3}$
4. $\dfrac{\$}{\text{dozen}} = \dfrac{4}{1}$
5. $\dfrac{\text{hours}}{\text{placemats}} = \dfrac{1}{8}$
6. $\dfrac{\text{miles}}{\text{hours}} = \dfrac{450}{1}$
7. $\dfrac{\text{miles}}{\text{gallon}} = \dfrac{20}{1}$

8. $\dfrac{\text{men}}{\text{women}} = \dfrac{3}{2}$
9. $\dfrac{\text{games lost}}{\text{games played}} = \dfrac{3}{13}$
10. $\dfrac{\$ \text{ materials}}{\$ \text{ labor}} = \dfrac{17}{60}$

Page 82
1.

cents	3	6	9	12	15	18	21
cans	2	4	6	8	10	12	14

2.

length	5	10	15	20	25	30	35
width	2	4	6	8	10	12	14

Page 83
1. proportion, 16, 112
2. proportion, 8, 312, 312
3. not a proportion, 15, 5, 45, 35
4. not a proportion, 36, 64, 9, 144, 576
5. proportion, 27, 11, 9, 297, 297
6. proportion, 3, 72, 2, 108, 216, 216
7. N
8. N
9. Y
10. Y
11. N
12. Y

Page 84
1. centimeters; $\dfrac{\text{cm}}{\text{ft}}$; $\dfrac{1}{3} = \dfrac{n}{18}$
2. yards;
   $\dfrac{\text{yards}}{\$}$; $\dfrac{2}{8.95} = \dfrac{n}{53.70}$
3. days; $\dfrac{\text{feet}}{\text{days}}$; $\dfrac{15}{1} = \dfrac{60}{n}$
4. calories;
   $\dfrac{\text{cookies}}{\text{calories}}$; $\dfrac{2}{270} = \dfrac{5}{n}$
5. miles; $\dfrac{\text{miles}}{\text{minutes}}$; $\dfrac{1}{3} = \dfrac{n}{5}$

6. pages; $\dfrac{\text{minutes}}{\text{pages}}$; $\dfrac{15}{1} = \dfrac{n}{25}$
7. scripts turned down; $\dfrac{\text{scripts turned down}}{\text{scripts accepted}}$; $\dfrac{25}{1} = \dfrac{n}{8}$
8. yellow triangles; $\dfrac{\text{red triangles}}{\text{yellow triangles}}$; $\dfrac{3}{9} = \dfrac{21}{n}$

Page 85
1. example
2. 8, 14, 168, 14, 12
3. n, 15, 3, 120, 40
4. $n = 32$
5. $n = 2$
6. $n = 12$
7. $n = 36$
8. $n = 18$
9. $n = 12$
10. $n = 21$
11. $n = 18$
12. $n = 20$
13. $n = 150$
14. $n = 15$
15. $n = 12$

Page 86
1. 7 tiles
2. 18 packages
3. $40
4. $31
5. 120 miles
6. 5 cards

Page 87
1. B
2. C
3. F
4. J
5. B
6. C
7. G
8. H