Properties of Exponents

An exponent (also called power or degree) tells us how many times the base will be multiplied by itself. For example x^5, the exponent is 5 and the base is x. This means that the variable x will be multiplied by itself 5 times. You can also think of this as x to the fifth power.

Below is a list of properties of exponents:

Properties	General Form	Application	Example
Product Rule *Same base add exponents*	$a^m a^n$	a^{m+n}	$x^5 x^3 = x^{5+3} = x^8$
Quotient Rule *Same base subtract exponents*	$\frac{a^m}{a^n}$	a^{m-n}	$\frac{x^9}{x^5} = x^{9-5} = x^4$
Power Rule I *Power raised to a power multiply exponents.*	$(a^m)^n$	a^{mn}	$(x^3)^4 = x^{3\cdot 4} = x^{12}$
Power Rule II *Product to power distribute to each base*	$(ab)^m$	$a^m b^m$	$(4x^3)^2 = 4^2 x^{3\cdot 2} = 16x^6$
Negative Exponent I *Flip and change sign to positive*	a^{-m}	$\frac{1}{a^m}$	$x^{-3} = \frac{1}{x^3}$
Negative Exponent II *Flip and change sign to positive*	$\frac{1}{a^{-m}}$	a^m	$\frac{1}{x^{-5}} = x^5$
Zero Exponent *Anything to the zero power (except 0) is one*	a^0	$a^0 = 1$	$(-4x)^0 = 1$

- It is important to note that none of these applications can occur if the bases are not the same.
  For example, $\frac{x^5}{y^3}$ cannot be simplified.

Using a Combination of Rules

At one point, you may be asked to use a combination of these properties.

Example:

- $\frac{(2^3y^2)^5}{2^{10}y^{16}}$ → Power Rule
- $\frac{2^{3\cdot5}y^{2\cdot5}}{2^{10}y^{16}}$
- $\frac{2^{15}y^{10}}{2^{10}y^{16}}$ → Quotient Rule
- $2^{15-10}y^{10-16}$
- 2^5y^{-6} → Negative Exponent
- $\frac{32}{y^6}$

Example:

- $\left(\frac{p^{-4}q}{r^{-3}}\right)^{-3}$ → Power Rule
- $\frac{p^{-4\cdot-3}q^{1\cdot-3}}{r^{-3\cdot-3}}$ ***Note:*** *When a base does not have an exponent there is really a one as the power. So that, q is understood as* q^1
- $\frac{p^{12}q^{-3}}{r^9}$ → Negative Exponents
- $\frac{p^{12}}{q^3r^9}$